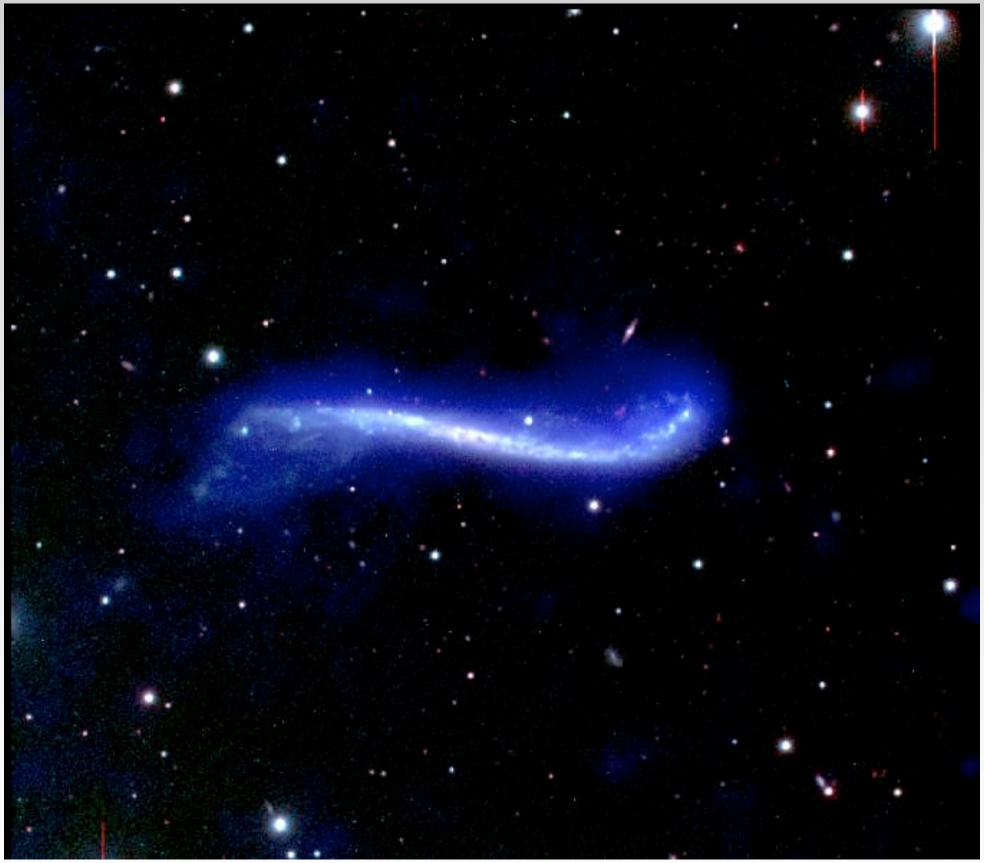
Imaging Concepts



Juan M. Usón (NRAO) 12/05/06

Luz

Ando pendiente de los juegos

de la luz

de como el vidrio empañado

se ilumina de repente

contrastando con la noche

de Alejandra Pinto

Light

I am taken by light's play,

how suddenly a fogged glass lights up

in contrast with the night

by Alejandra Pinto

Introduction: Important concepts

Radio interferometers are linear devices

Imaging: Estimation of true sky brightness from the observed visibilities

Imaging is a non-linear process

① Imaging: Fourier inversion of the visibilities

Weighting modifies the point-spread function
and the noise characteristics (SNR)

② Deconvolution: Correcting for "missed" visibilities

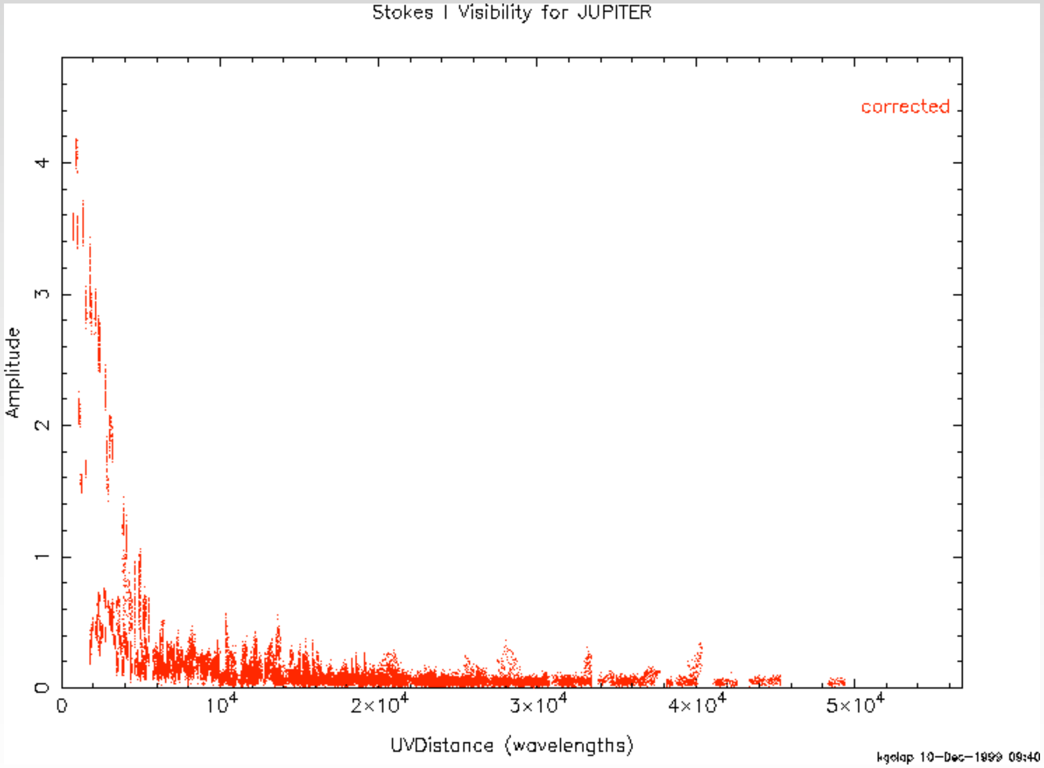
A number of methods lead to somewhat different results

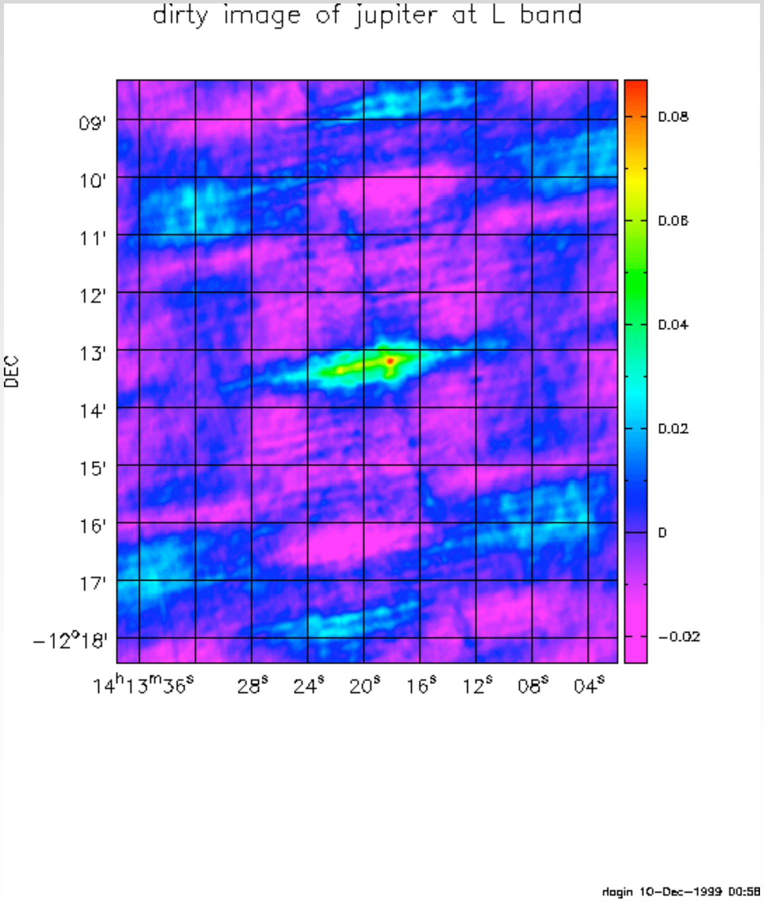
③ Self-calibration: Correcting the visibilities to sharpen the image

Improve on calibration (SNR permitting)

Imaging

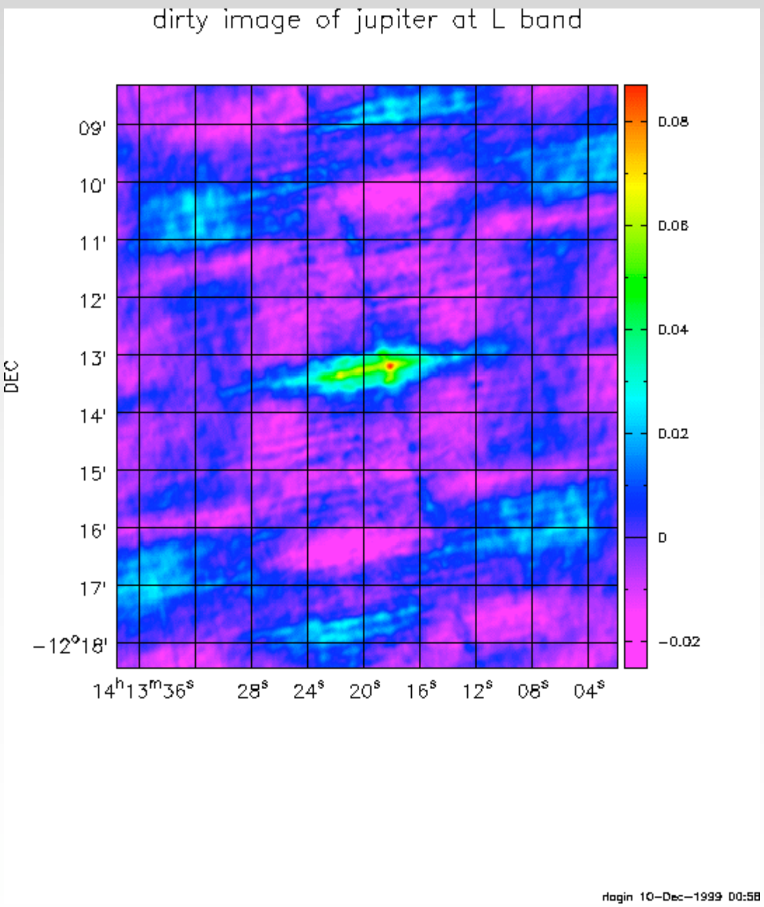
- Go from samples of the visibility function to the “dirty” image

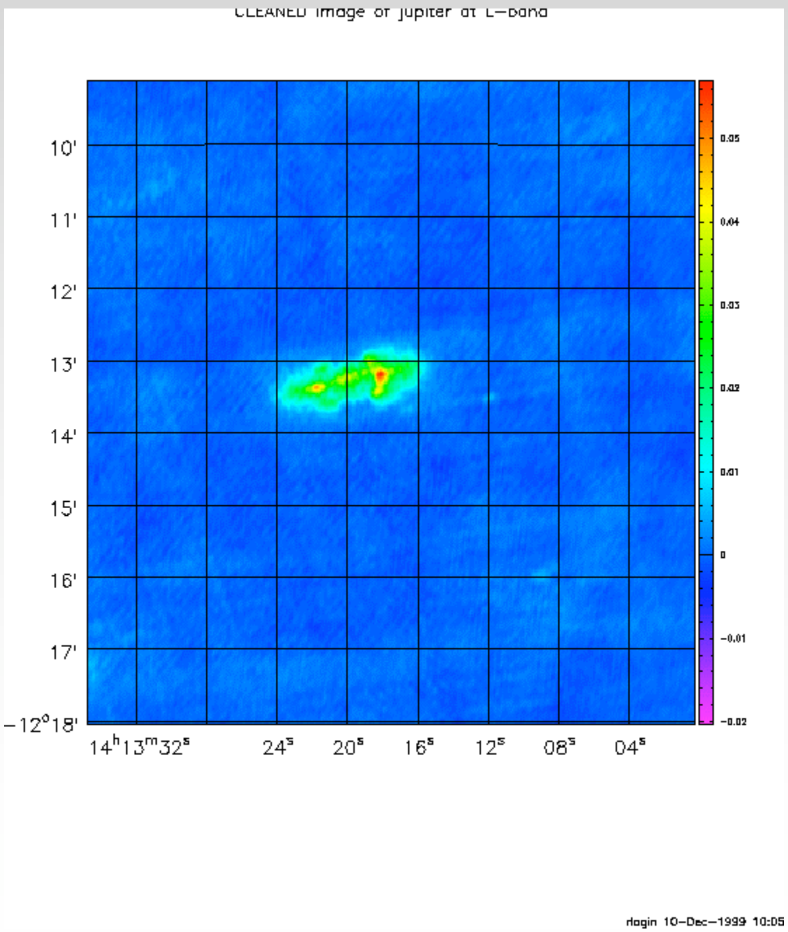




Deconvolution

- Go from dirty image to deconvolved image

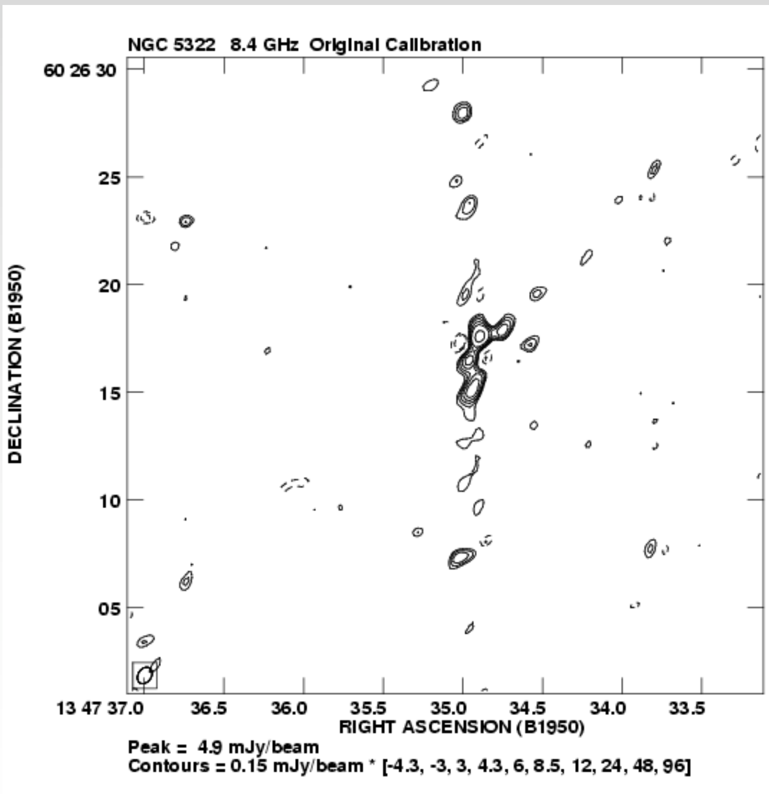




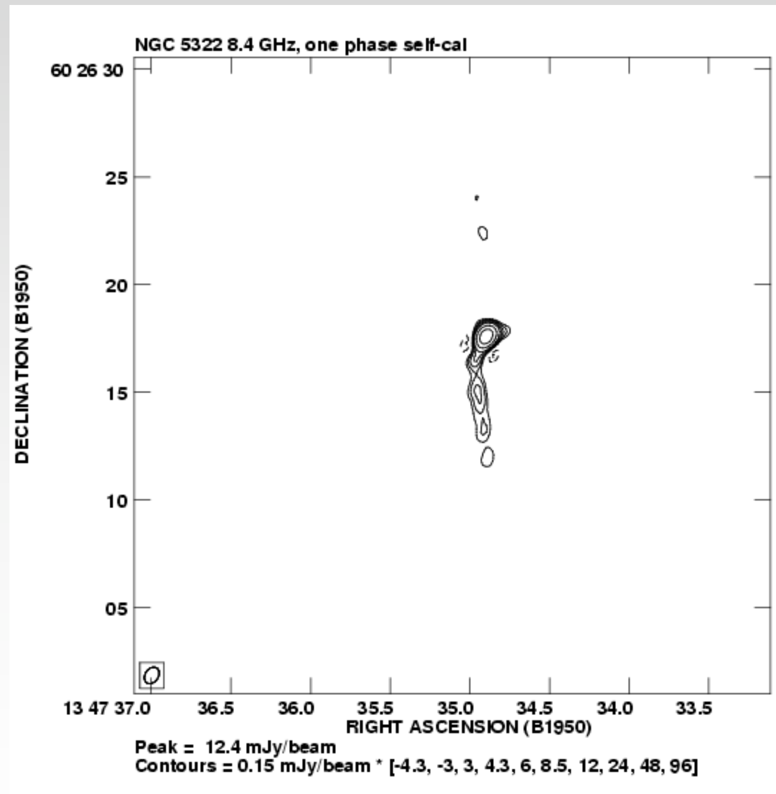
Self-Calibration

Sharpen the deconvolved image

- First deconvolved image



Self-calibrated image



Outline

- The relationship between sky brightness and visibility
- Sampling of the Fourier plane
- Fourier inversion
  - Weighting schemes
  - The problem with the dirty image - sidelobes
- Deconvolution
  - CLEAN
  - Maximum Entropy Method
  - Algebraic deconvolution
  - Other methods
- Some examples

Formal description

- For small fields of view, the visibility function is the 2-D Fourier transform of the sky brightness:

$$V(u,v) = \int I(l,m) \cdot e^{j.2\pi.(ul+vm)} dl.dm$$

- We sample the Fourier plane at a discrete number of points:

$$S(u,v) = \sum_k w_k \cdot \delta(u-u_k) \cdot \delta(v-v_k)$$

- So the inverse transform is:

$$I^D(x,y) = F^{-1}[S(u,v) \cdot V(u,v)]$$

- Applying the Fourier convolution theorem:

$$I^D(x,y) = B(x,y) \otimes I(x,y)$$

- where B is the point spread function:

$$B(x,y) = F^{-1}[S(u,v)]$$

Convolution theorem

- Inverse Fourier transform of the sampled visibilities yields the true sky convolved with the point spread function (PSF)
- Different ways to understand this effect:
  - Incomplete Fourier sampling => missing information about the sky brightness
  - Array ≡ masked aperture => diffraction patterns in image plane

To find the true sky brightness I, we must "deconvolve" the point spread function B from the dirty image I^D.

- What are the properties of the point spread function?
  - "sidelobes" with infinite extent
  - Invisible distributions

A digression: Fast Fourier Transforms

- FFTs are much faster than simple Fourier summation but a regular gridding is required
- Visibility data are irregularly sampled so we must resample the data on a regular grid
- Convolutional gridding is used: the discrete visibility samples are notionally smoothed to a continuous function, and then resampled at the regular grid points.
- Time-consuming but generally worthwhile
- Some fraction of the power is applied to the incorrect spatial frequencies: *aliasing* or *spurious sources*, usually at a very low level
- Long description in *Synthesis Imaging II*

Some Fourier transform pairs

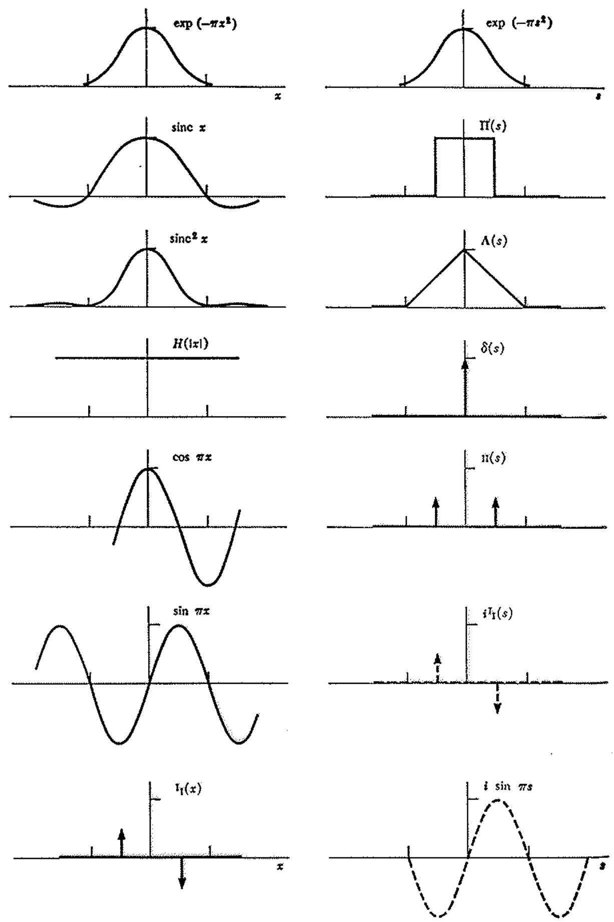


Fig. 6.1 Some Fourier transform pairs for reference.

Sidelobes

- From the sampling pattern, we can find that:

$$B(x,y) = \frac{1}{K}\sum_k \cos(u_k x + v_k y)$$

- So the point spread function is always a collection of co-sinusoids that extends throughout the image plane
- At the center, $B(x,y) \sim 1$
- The PSF has a width $\Delta x \sim 1/u_{\max}$, $\Delta y \sim 1/v_{\max}$
- The RMS level is ~ $1/\sqrt{K}$

More on sidelobes

- Far-out sidelobes:

  From the Fourier derivative theorem, if the sampling pattern has a discontinuous first derivative, the PSF drops off as the inverse of the radius in the image plane

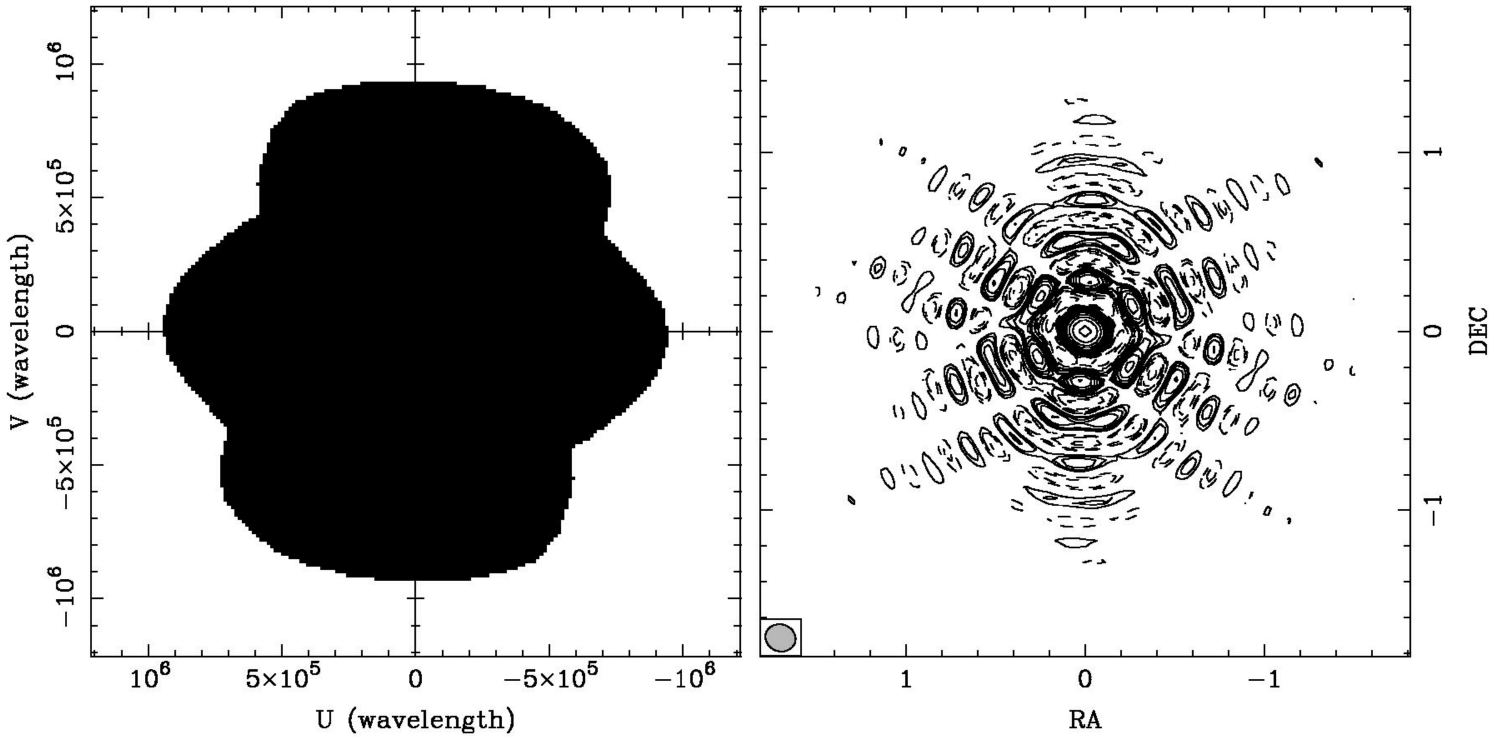
- Close-in sidelobes:

  Suppose that the sampling pattern is bounded by a circle, then the PSF close in must resemble the inverse Fourier transform of a circle: first order Bessel function divided by radius: *Jinc* function

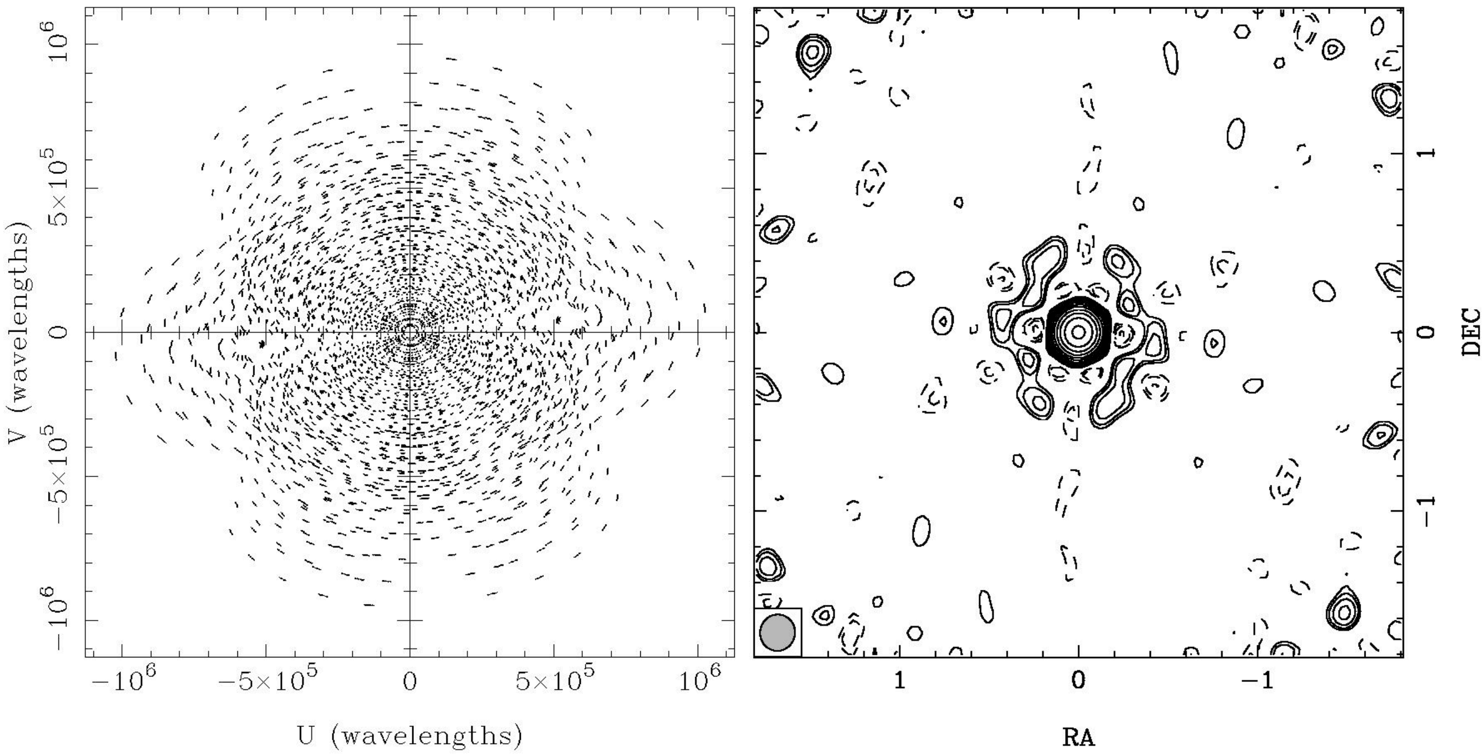
- Can apply weighting to ameliorate these two effects:

$$B(x,y) = \frac{\sum_k w_k \cos(u_k x + v_k y)}{\sum_k w_k}$$

Close-in sidelobes



Close-in sidelobes



Weighting

- Choose the weighting function to alter properties of PSF:
- Uniform weighting
  - To minimize RMS sidelobes over entire image requires:

$$w_k = 1/\rho(u_k, v_k)$$

  - But SNR suffers...
- Natural weighting
  - To minimize noise over entire image requires:

$$w_k = 1/\sigma_k^2$$

- Briggs (robust) weighting
  - To minimize noise plus sidelobes for point source of strength S requires

$$w_k = 1/\left[S^2 \cdot \rho(u_k, v_k) + \sigma_k^2\right]$$

More on weighting

- <u>Super-uniform weighting</u>
  - Can choose to minimize sidelobes over smaller region than entire image
  - Divide out density averaged over large region in Fourier space
- All weighting decreases the sensitivity relative to natural weighting
- Uniform weighting increases the resolution relative to natural weighting
- Briggs' weighting allows a compromise between sensitivity and resolution

An Example:

- Observations of Hydra A with the VLA in B configuration (data courtesy of Greg Taylor (University of New Mexico)
- Location Of VLA Antennas

```
                          N36 ( 2)
                         N32 (27)
                        N28 ( 8)
                        N24 ( 3)
                       N20 (15)
                       N12 ( 6)
                       N8  ( 5)          VLA:OUT  (13), (20)
                      N4  (17)
                 (28) W4     E4  (18)
              ( 9) W8            E8  (23)
          ( 1) W12                  E12 (16)
        (21) W16                       E16 (22)
      (12) W20                            E20 (26)
    ( 4) W24                                 E24 (24)
  (11) W28                                      E28 (14)
 (19) W32                                          E32 (25)
(10) W36                                              E36 ( 7)
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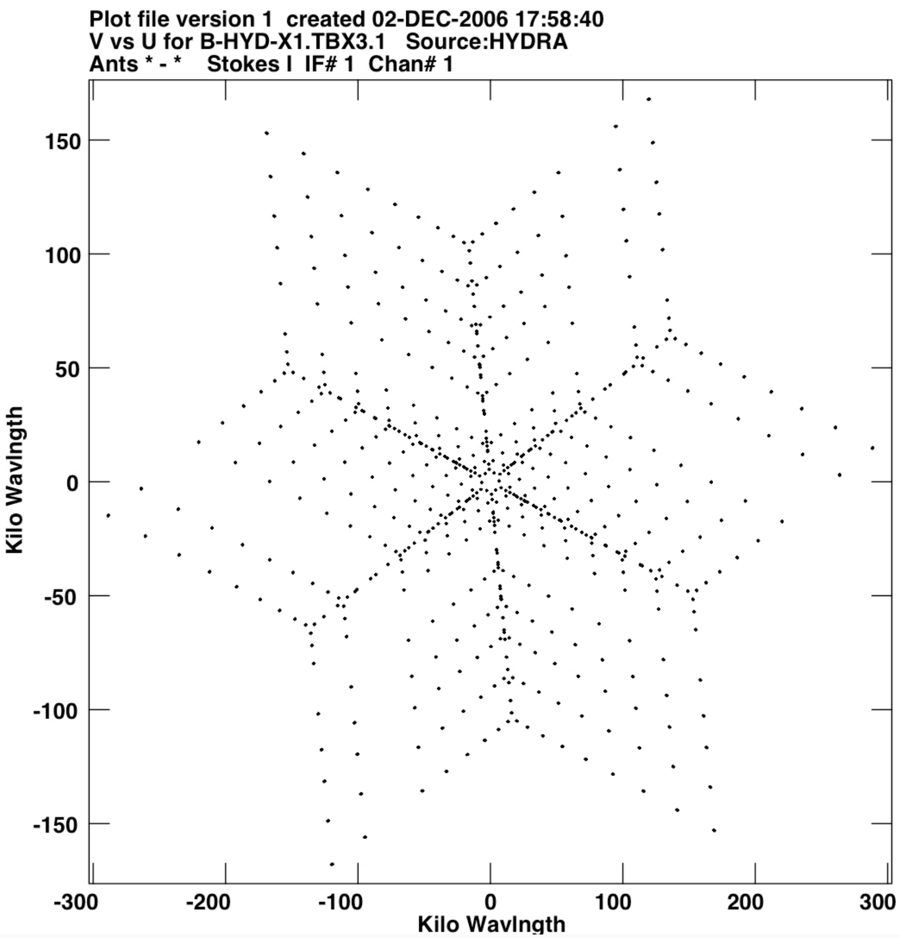
uv-coverage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Ants * - * Stokes I IF# 1 Chan# 1
Kilo Wavlngth
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100
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-200
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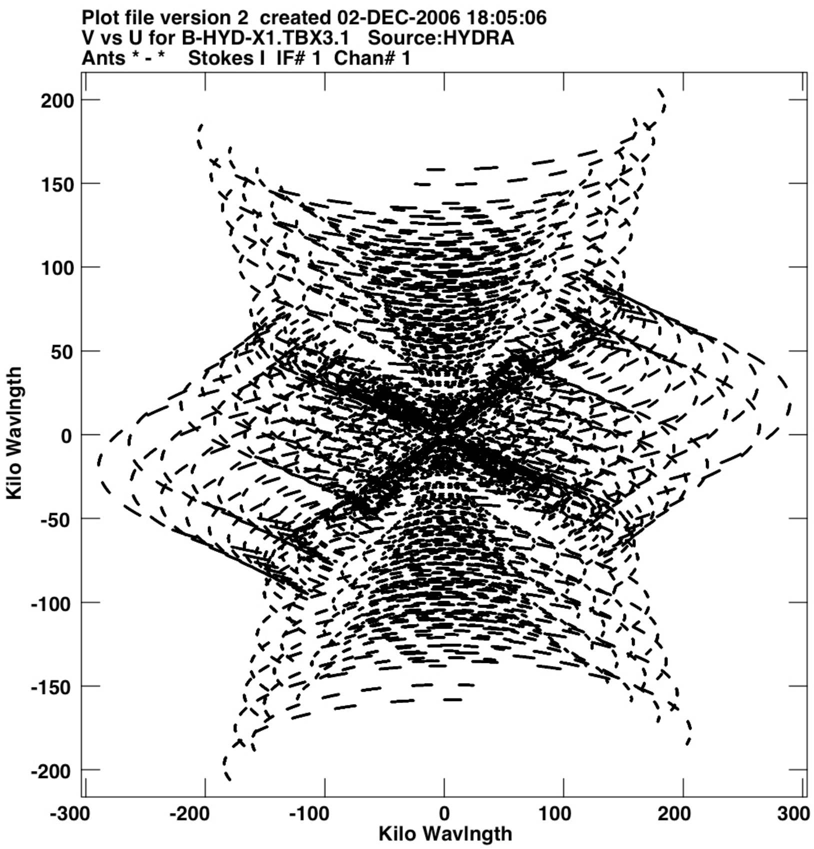
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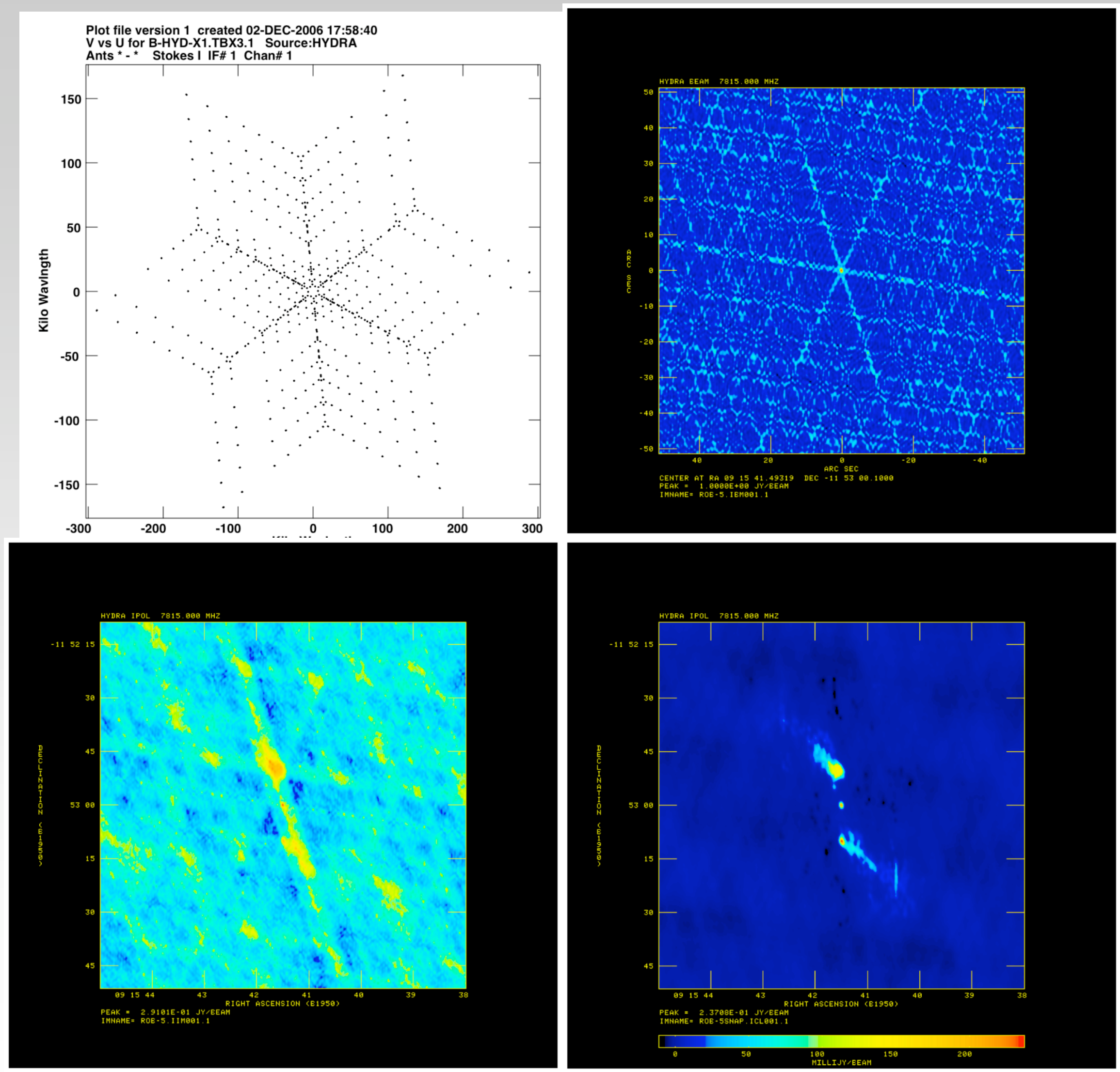
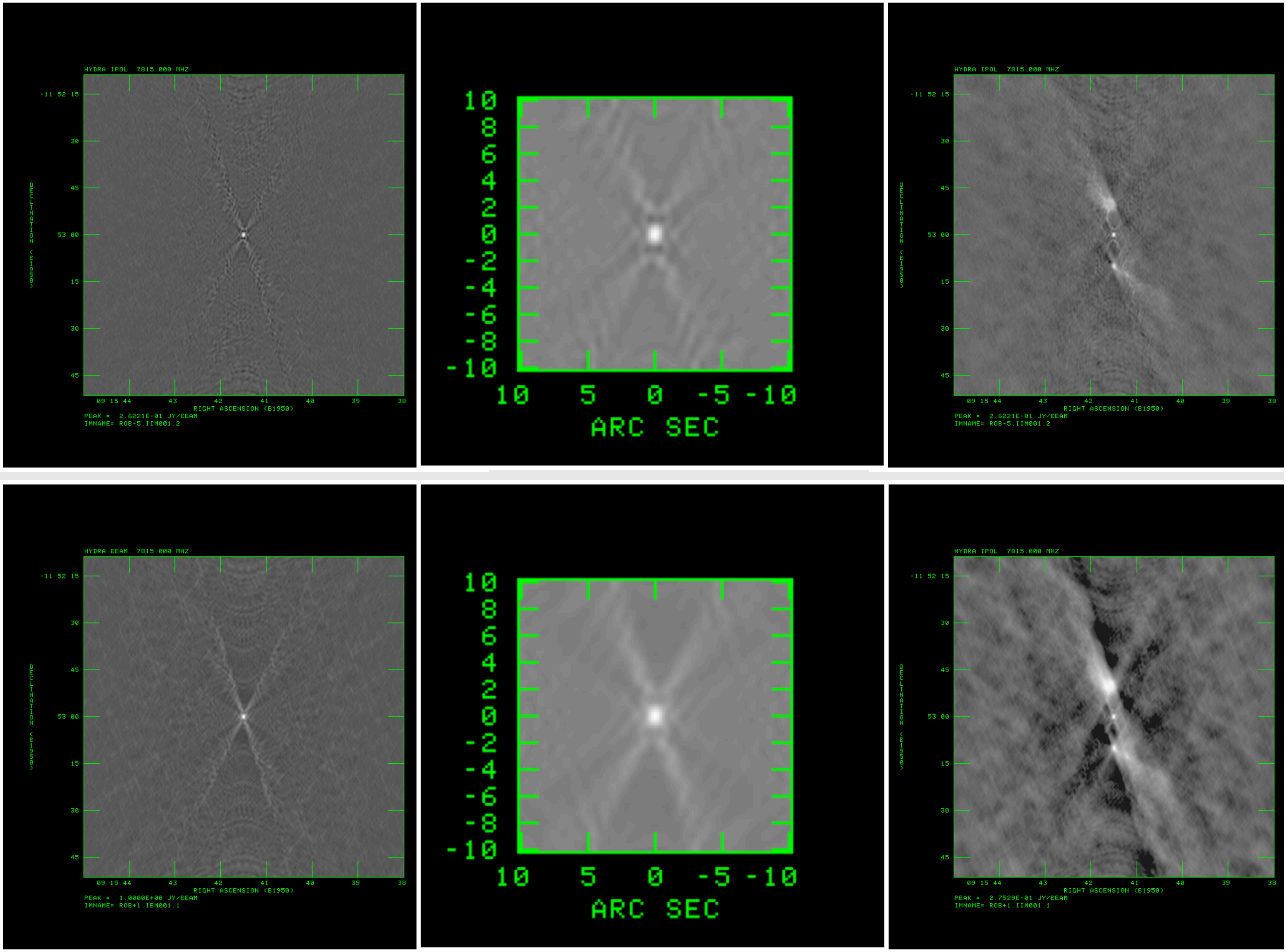
Ants * - * Stokes I IF# 1 Chan# 1
Kilo Wavlngth
Kilo Wavlngth
200
150
100
50
0
-50
-100
-150
-200
-300
-200
-100
0
100
200
300

“Snapshot” image

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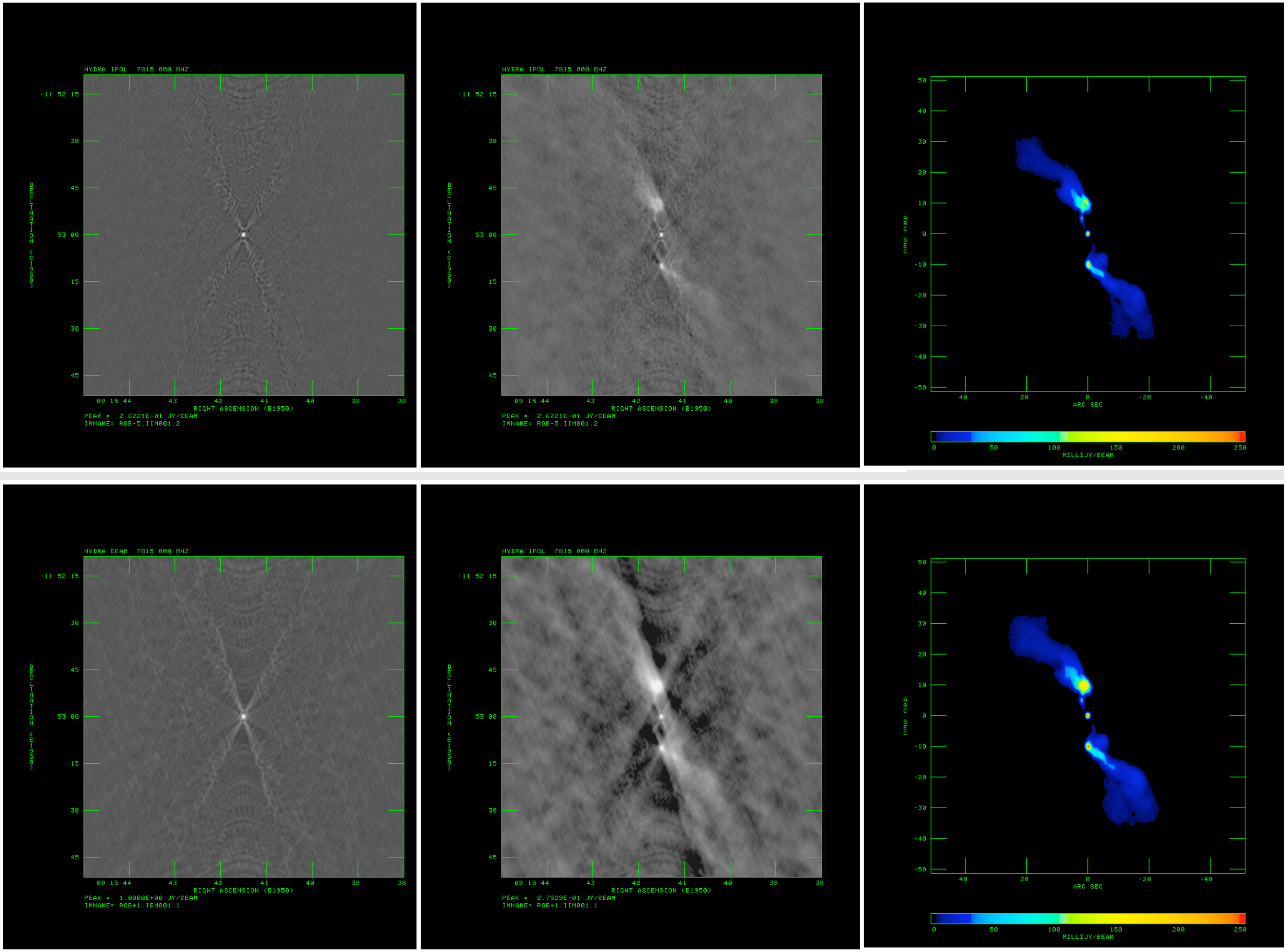
RIGHT ASCENSION (B1950)
DECLINATION (B1950)
PEAK = 2.6221E-01 JY/BEAM
ARC SEC
HYDRA BEAM 7815.000 MHZ
PEAK = 1.0000E+00 JY/BEAM
PEAK = 2.7529E-01 JY/BEAM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ARC SEC
ARC SEC
MILLIJY/BEAM

Tapering

- Can go further, and multiply by a desired sampling shape:

$$w_k = T(u,v)/\rho(u_k, v_k)$$

- For example, the desired shape could be a Gaussian, which transforms to a Gaussian, and therefore falls away rapidly in the image plane
- BUT, the underlying sampling pattern eventually wins...

  Weighting and tapering help, but cannot entirely remedy the limitations in the image due to finite Fourier plane sampling

Invisible distributions

- There are sky brightness distributions Z that are invisible:

$$B \otimes Z = 0$$

- This occurs when the spatial frequencies (u, v) in the invisible distribution Z are not sampled
- Some examples:
  - Total integrated brightness (usually but not always)
  - Short spacings below the *minimum* separation of antennas
  - Long spacings beyond the *maximum* separation of antennas
  - Holes in the sampling pattern
  - Any combination of the above!
- No *linear* method can ever recover the invisible distributions

$$D \otimes I^D = D \otimes [B \otimes (I + Z)] = D \otimes B \otimes I$$

How can we determine the invisible distributions?

- Apply *a priori* knowledge about the sky brightness
- What do we know?
  - Sky brightness is positive, sum of co-sinusoids is not necessarily
  - Sky is mostly dark, sum of co-sinusoids is not
  - Sky is collection of point sources, sum of co-sinusoids is not
  - Sky may be smooth, sum of co-sinusoids is probably not
- *Non-linear* deconvolution algorithms solve for an estimate of the true sky brightness I, from the convolution equation, while applying *a priori* constraints on the final solution

Popular deconvolution algorithms

- CLEAN:
  - sky is composed of point sources on a dark sky
  - sky is composed of resolved sources of known extent on a dark sky
- Multi-scale CLEAN:
  - sky is composed of smooth, limited extent blobs on a dark sky
- Maximum Entropy Method:
  - sky is smooth and positive
- Non-negative least squares:
  - sky is non-negative and compact
- Hybrid algorithms:
  - Some combination of the above...

Classic CLEAN

- *A priori* constraint: sky is composed of point sources on a dark sky
- Uses iterative algorithm to find sequence of point sources
  - Find peak in image
  - Subtract a PSF centered and scaled appropriately to remove the effect of the brightness point, store component thus found
  - If any significant points left, return to first step
  - Convolve point components by "Clean" point spread function
    - Same width as dirty PSF but no sidelobes
  - Add residuals image to obtain "restored" image
- Classic CLEAN algorithm due to Högbom (1974)

Classic CLEAN details

- Usually stabilize algorithm by subtracting only a fraction (the loop gain ~ 0.1) of the strength of a point source
- Usually stop either after finding a given number of components or when the peak residual is reaches a threshold, such as a multiple of the intrinsic noise level
- Schwarz (1978) showed that
  - Classic CLEAN must converge *i.e.* the peak residual must decrease
  - Classic CLEAN is equivalent to a least square fit of sinusoids to the visibility data
- Excellent at reducing identifying and correcting for point sources, less effective for extended emission in neighboring pixels

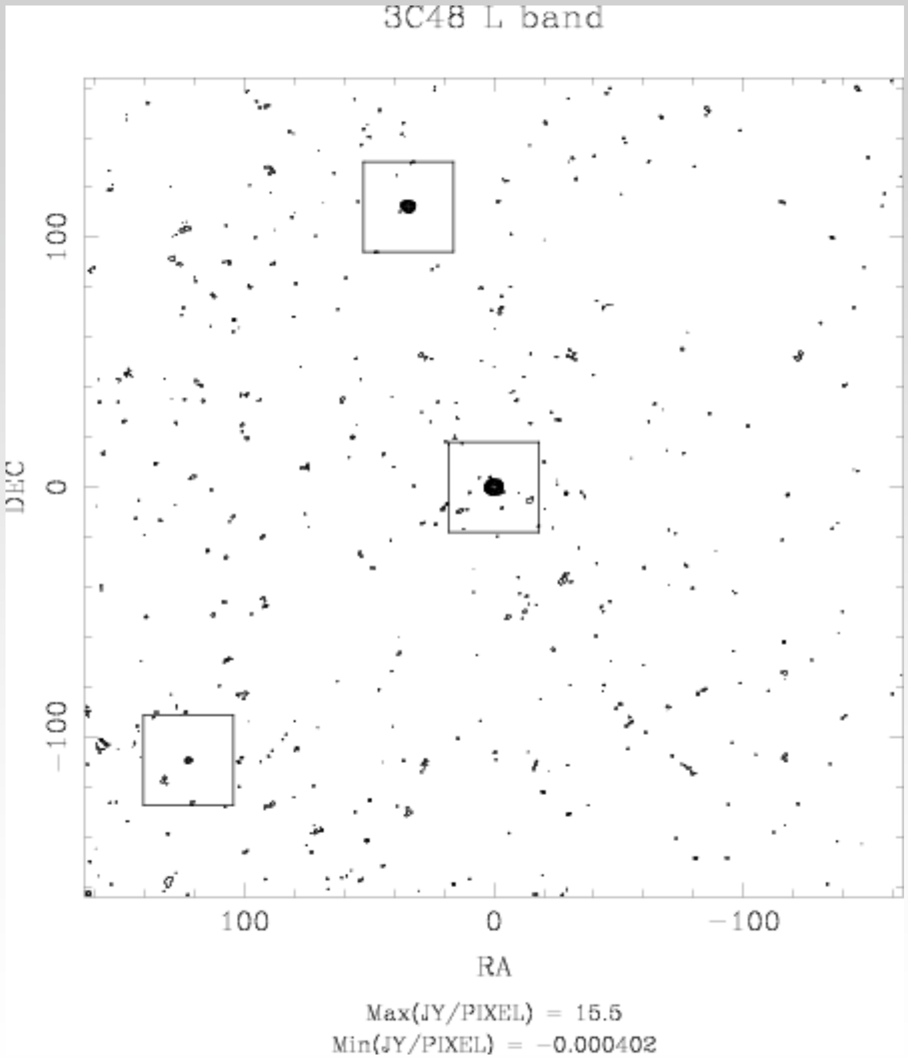
Classic Window CLEAN

- *A priori* constraint: sky brightness extent is known a priori
- Uses Classic CLEAN iterative algorithm to find sequence of point sources in restricted region delimited by CLEAN boxes
- Allows close specification of source support constraints
- Very useful for poor Fourier plane coverage *e.g.* VLBI

CLEAN variants

- Clark CLEAN: faster variant of Högbom CLEAN
  - Split into two stages
  - Cleans subset of brightness points in minor cycle
  - Subtracts sidelobes completely using Fast Fourier Transform convolution in major cycle
  - 0.1-10 times faster than Högbom
- Schwab-Cotton CLEAN: another variant of Clark CLEAN
  - Clark minor cycle
  - Major cycle subtracts components directly from visibility data
  - Sometimes faster, always more accurate than Clark CLEAN
  - Can clean multiple fields
- Steer-Dewdney-Ito: variant of Clark CLEAN
  - Minor cycle simply takes scaled version of pixels brighter than some trim level

Schwab-Cotton CLEAN



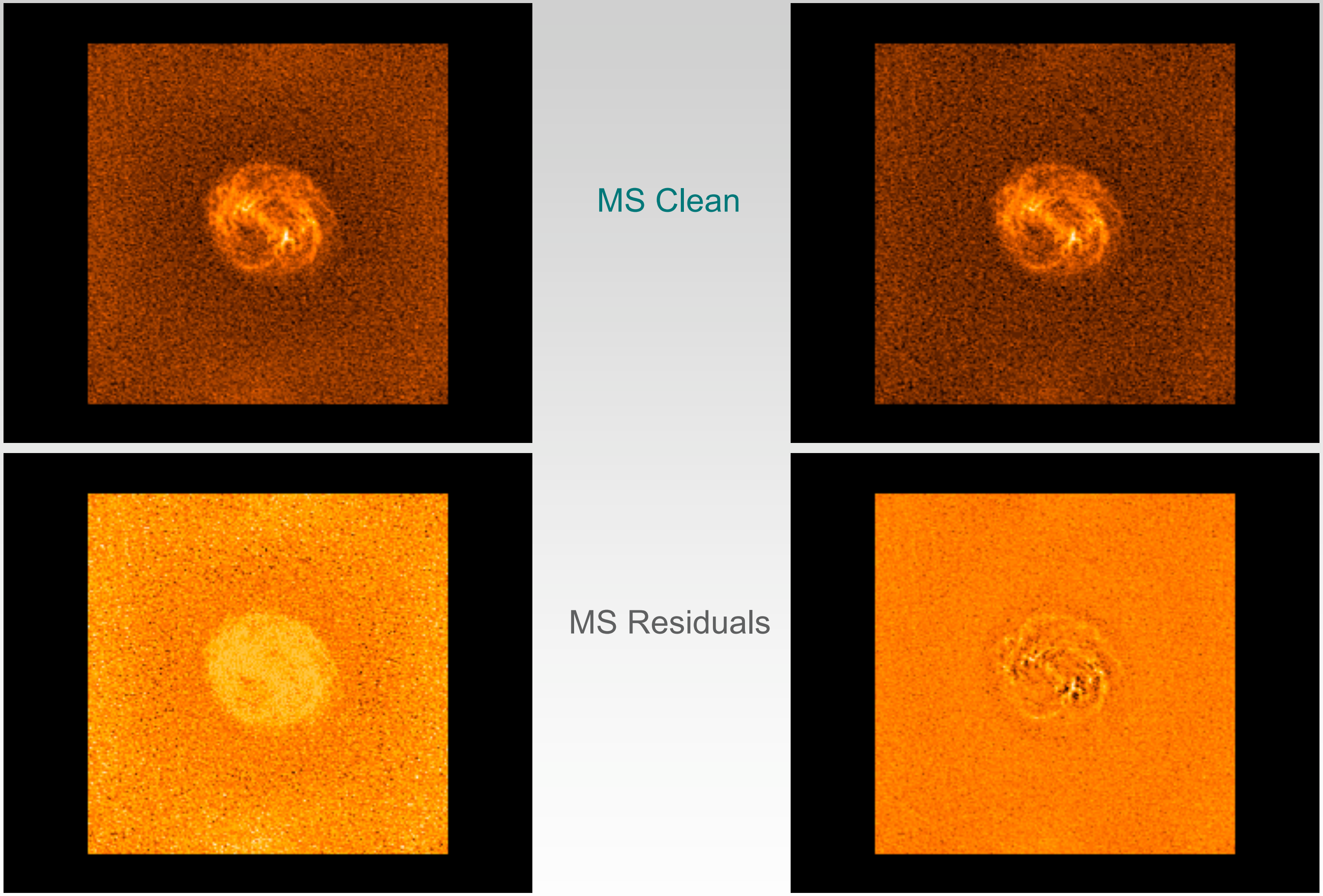
Multi-scale CLEAN

- *A priori* constraint: sky is composed of smooth blobs on a dark background
- Decompose sky into summation of blobs of various sizes *e.g.* truncated parabolas of width 0, 3, 10, 30 pixels.
- Perform one CLEAN algorithm for each scale size in parallel, and choose blob that gives the greatest reduction in peak residual
- Excellent at identifying large-scale coherent structure
- Residuals are quite noise-like

Multi-scale CLEAN

Clark Clean

MS Clean



Clark Residuals

MS Residuals

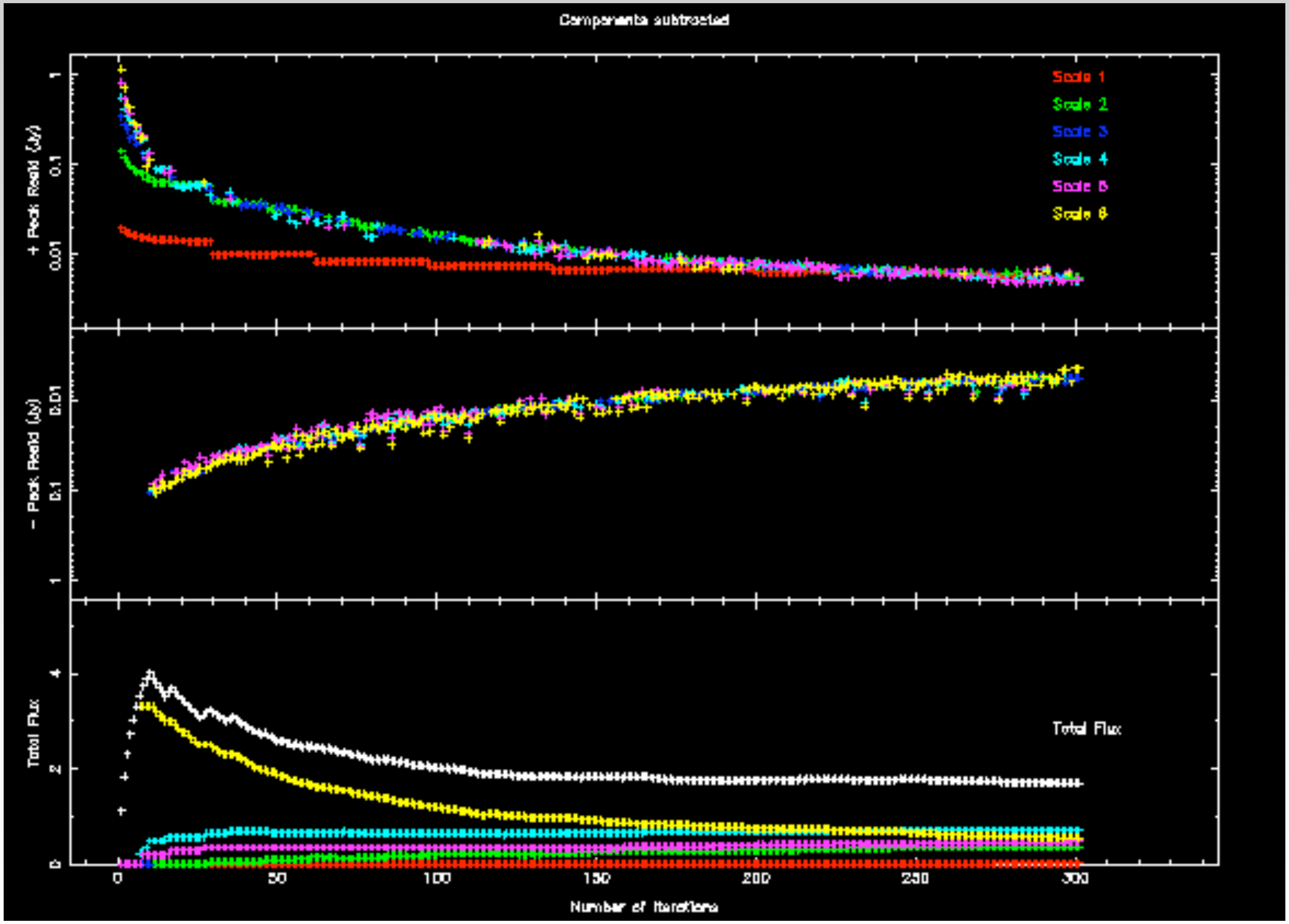
Multi-scale CLEAN

Convergence

on various

scale sizes.

Clean gain=0.5



Maximum Entropy Method

- *A priori* constraint: sky is smooth and positive
- Algorithm maximize a measure of smoothness (entropy) while solving the convolution equation

$$H(I) = -\sum_k I_k \cdot \log(I_k / m_k)$$

- where m_k is a "default" image which is the image obtained with no data. Usually a flat default image is used.
- Non-linear optimization problem: Cornwell-Evans (1983)
- Excellent for large diffuse emission
- Default image is very powerful for incorporating prior images
- Extensible to multiple simultaneous convolution equations (applicable to mosaicing)

Maximum Entropy Method details

- Fast and efficient for million or more pixels
- Excellent on smooth extended emission with limited dynamic range
- Point sources cause problems
  - Should be removed using CLEAN before applying MEM
- Much has been written about philosophy and meaning of MEM

Algebraic deconvolution

- Pixellate convolution equations and represent via linear algebra $A\underline{x}=\underline{b}$ where the matrix A represents the point spread function, $\underline{x}$ is the unknown image as a vector, and $\underline{b}$ is the dirty image as a vector.
- This linear equation must be solved using various constraints
  - *e.g.* support constraints: we know that the emission is non-zero for only some areas
- Solve equation using *e.g.* Singular Value Decomposition
  - usually inadequate to get reasonable result but useful as indication of conditioning of the problem

Non-Negative Least Squares

- Impose non-negativity using any of a variety of solvers

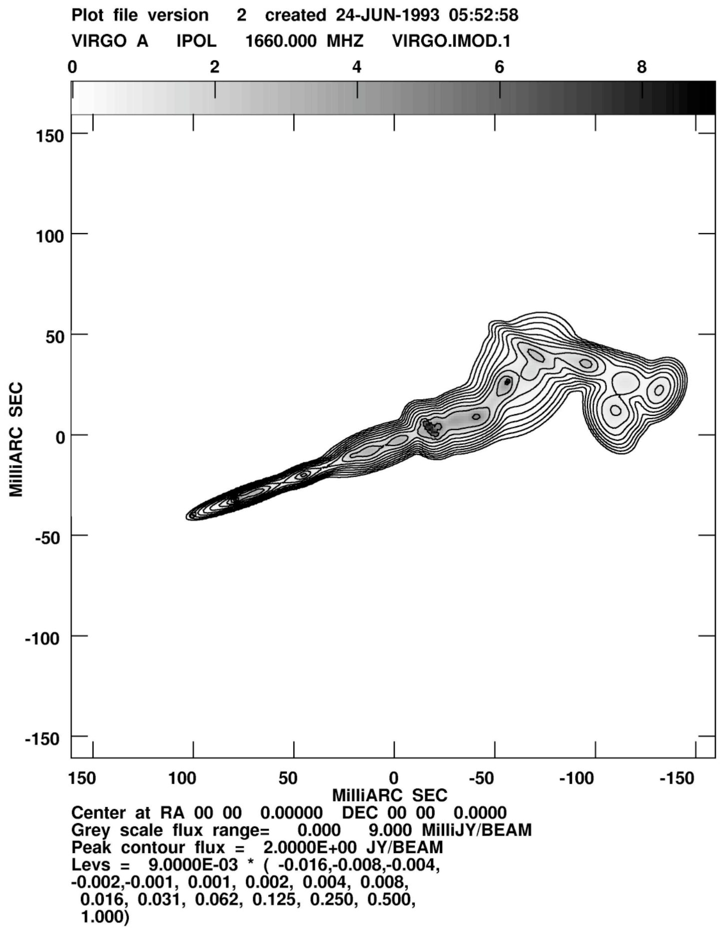
  Solve $A\underline{x} = \underline{b}$ subject to $\underline{x} \geq 0$

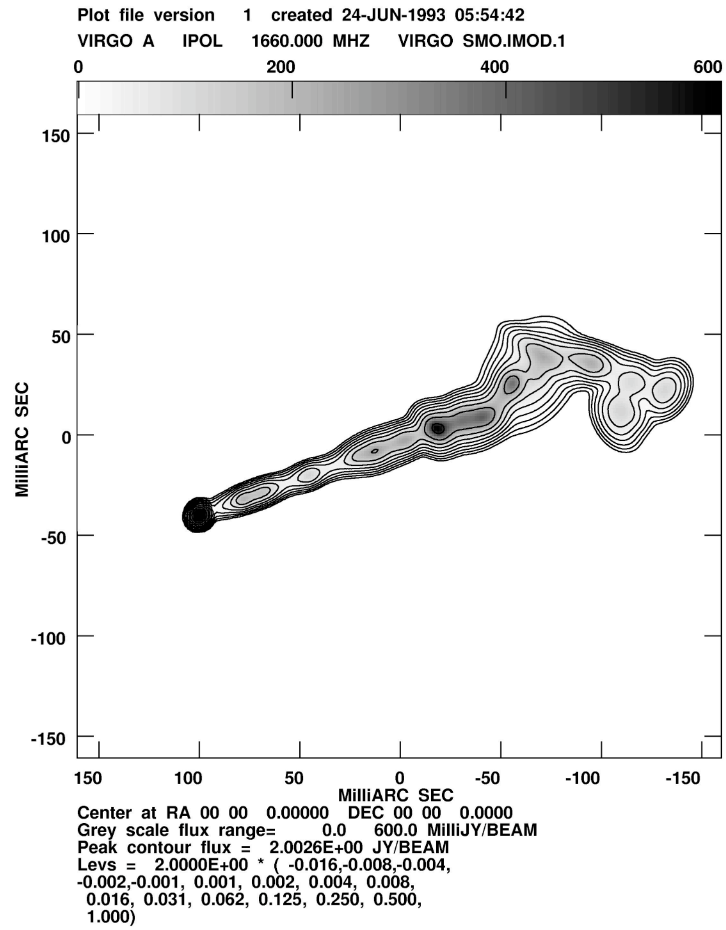
- Works well for high dynamic range images of moderately resolved sources (Briggs' thesis, 1995)

Example

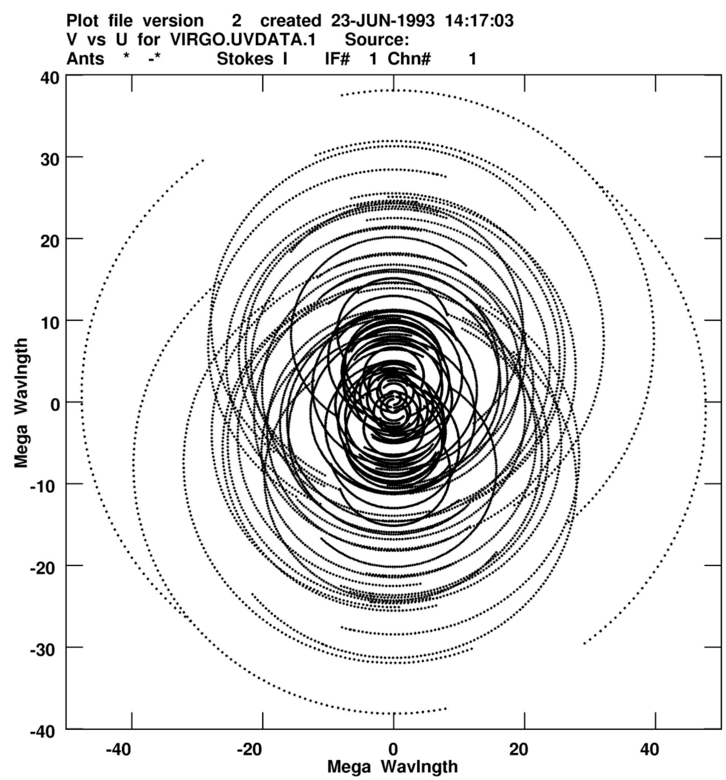
- VLBA simulated observations of M87-like jet source
- Will show
  - uv coverage
  - Visibility function
  - Point Spread Function
  - Dirty image
  - Clean images
  - Maximum Entropy images

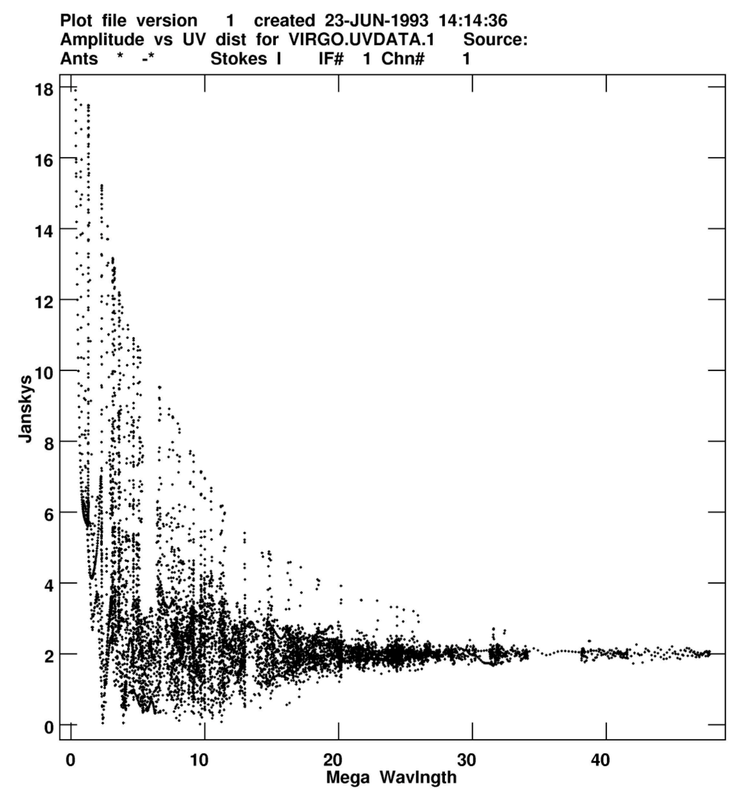
Original and smoothed model



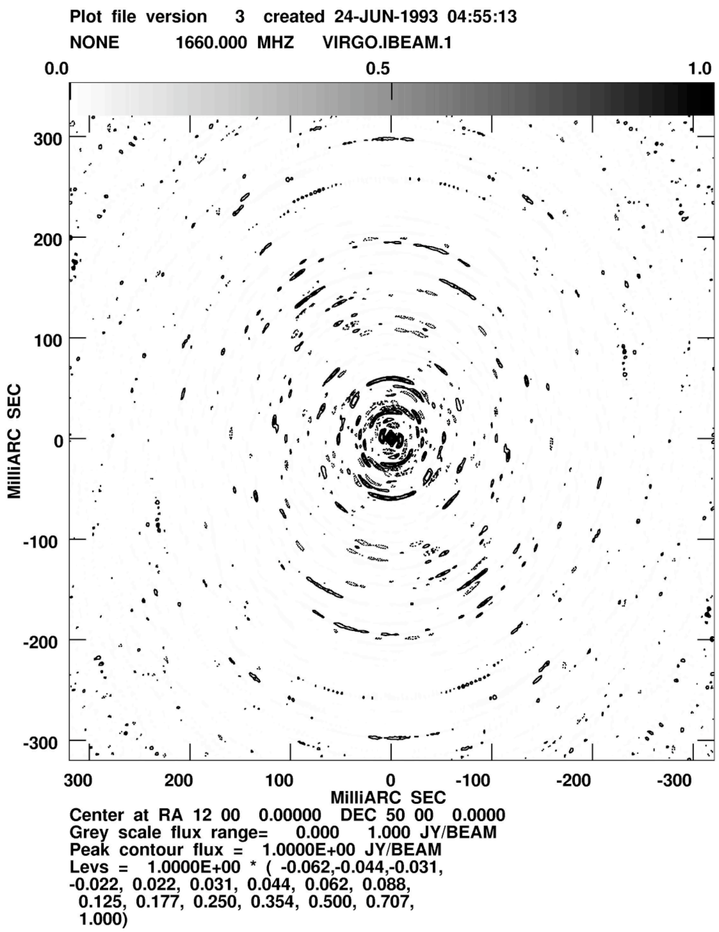


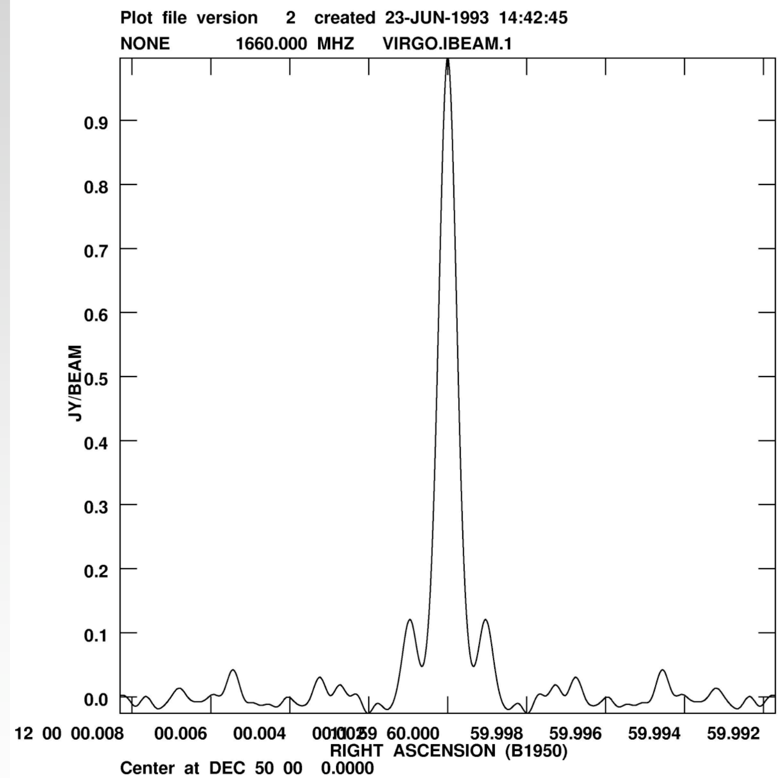
Fourier plane sampling



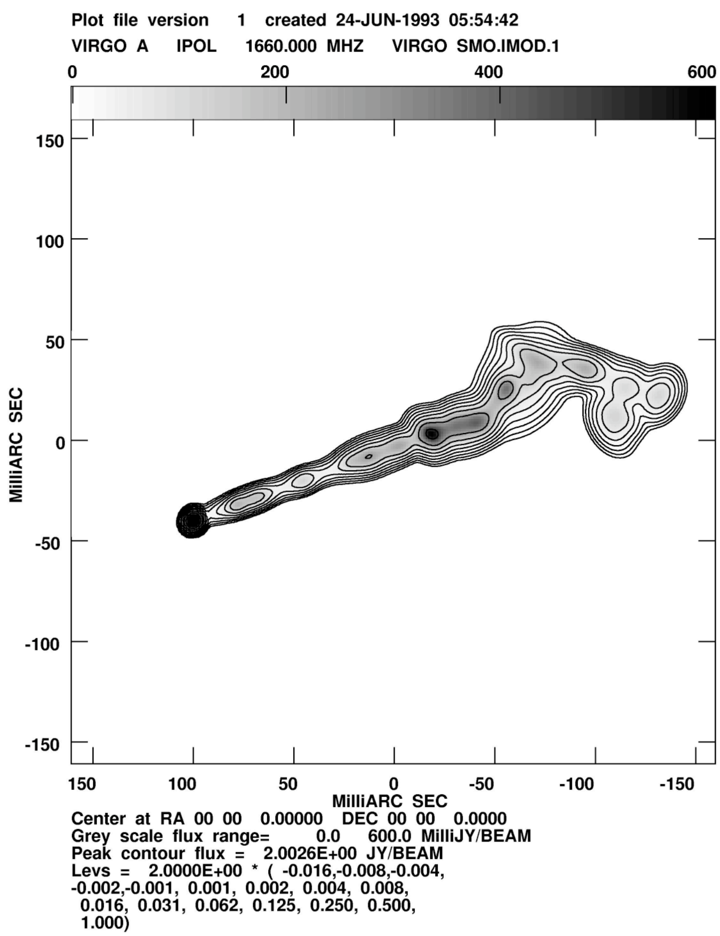


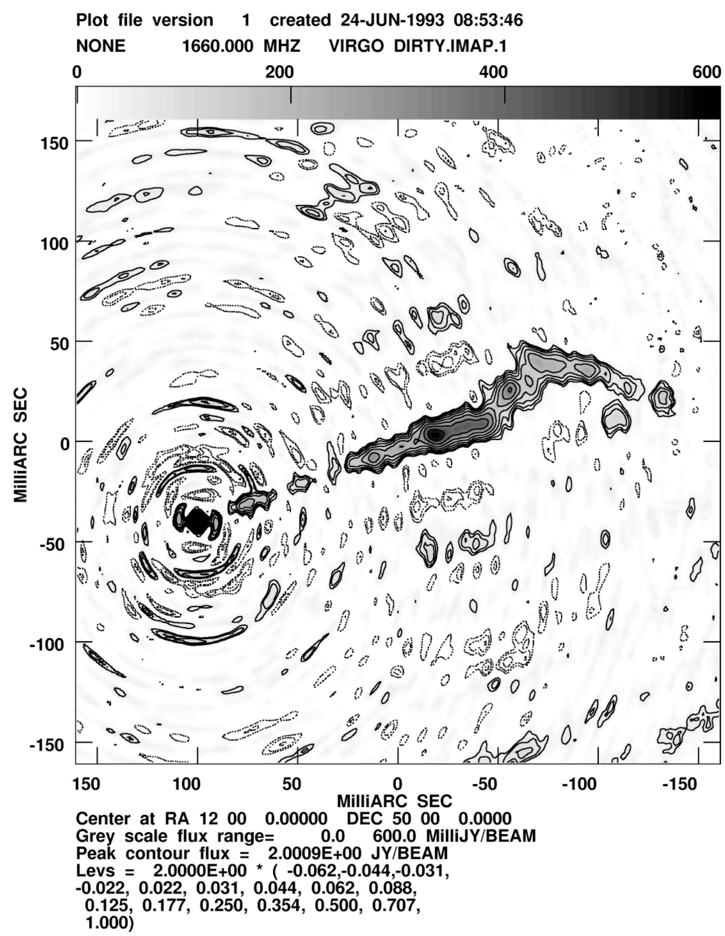
Point Spread Function



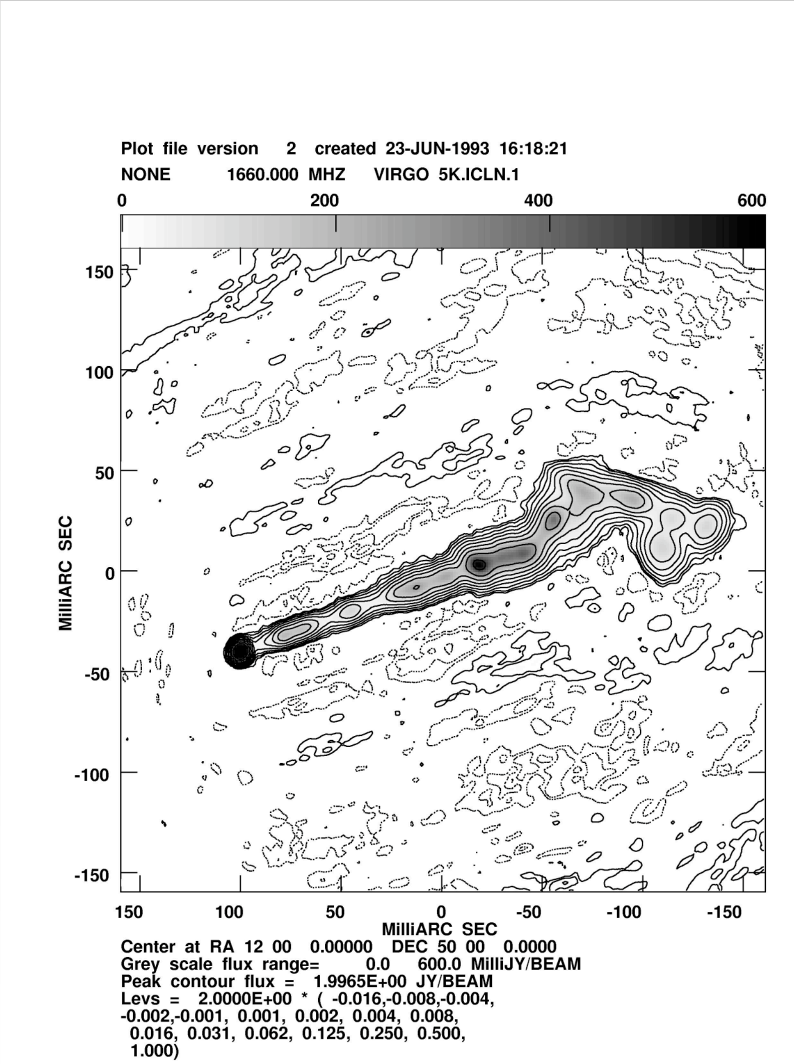


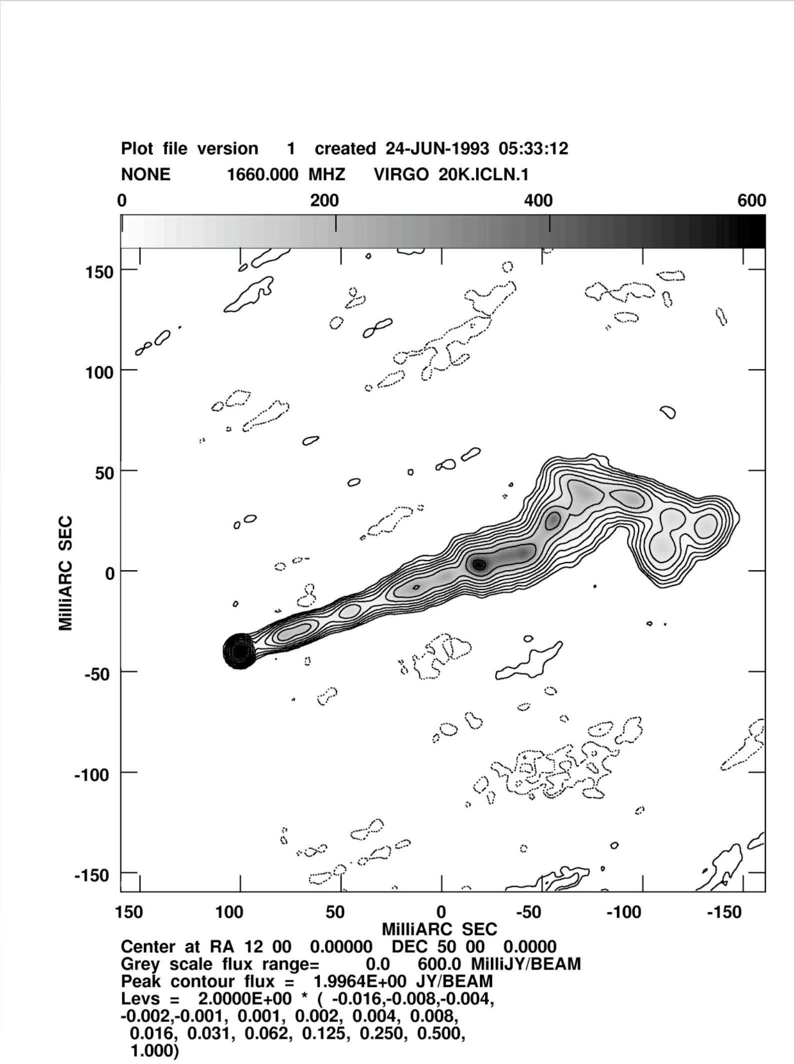
Original model and Dirty image



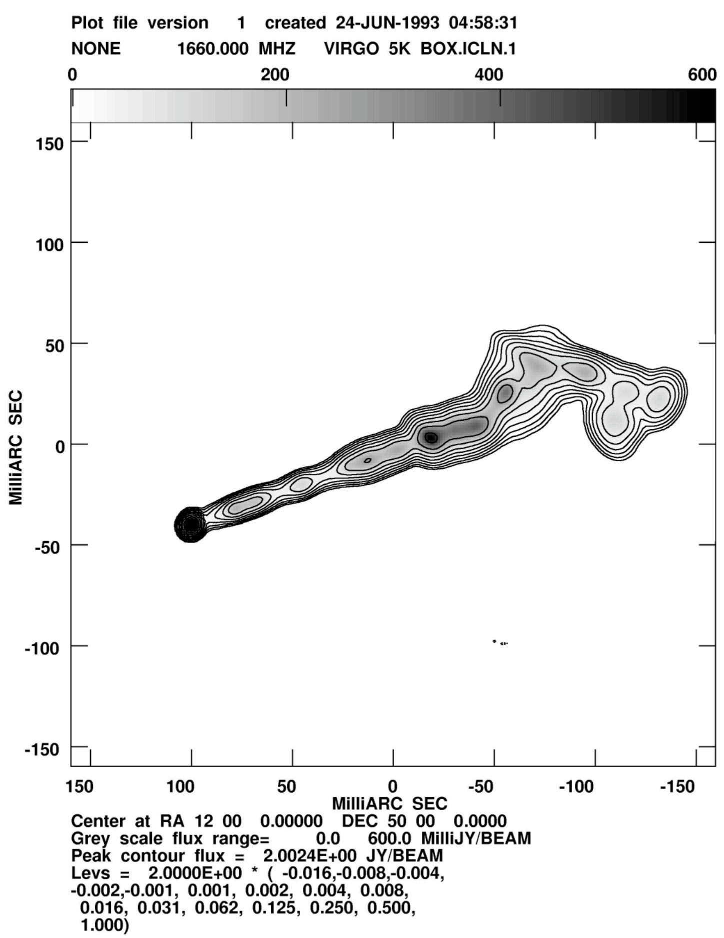


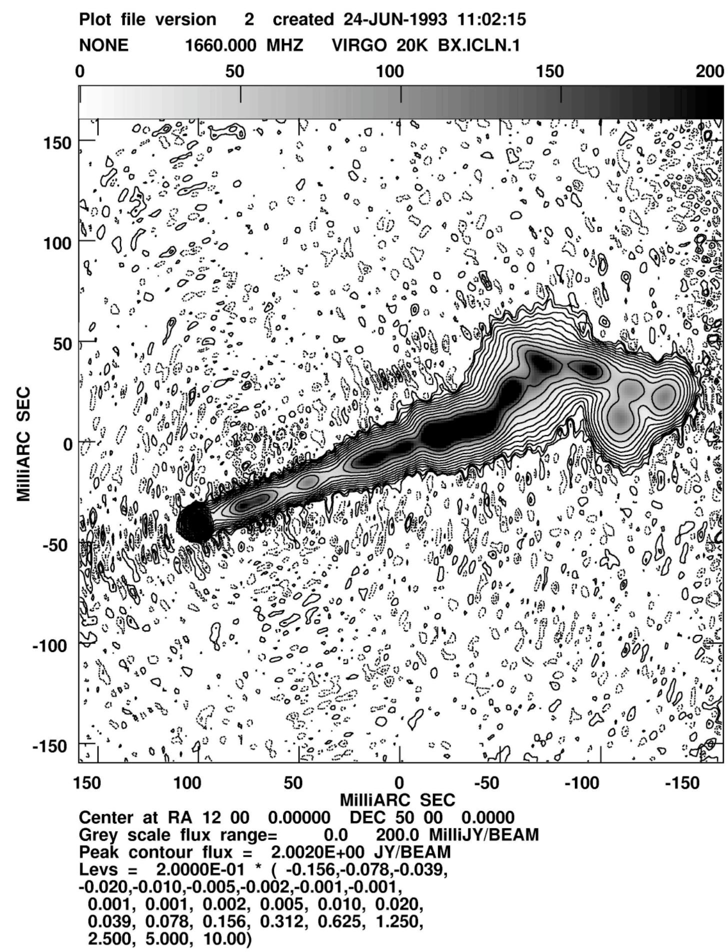
Classic CLEAN: 5000 and 20000 comps



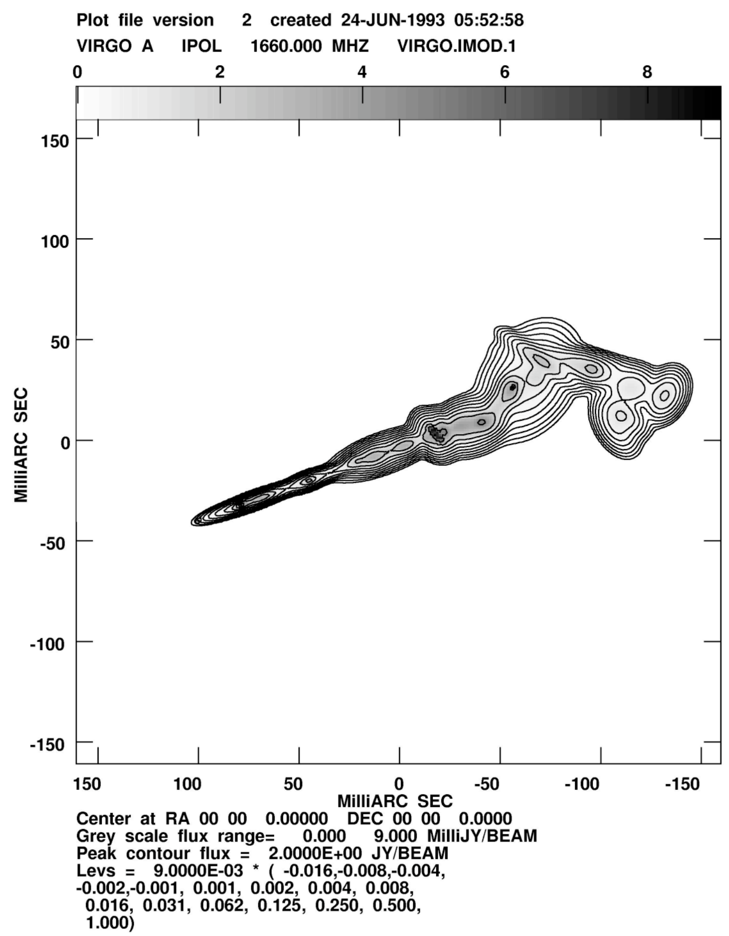


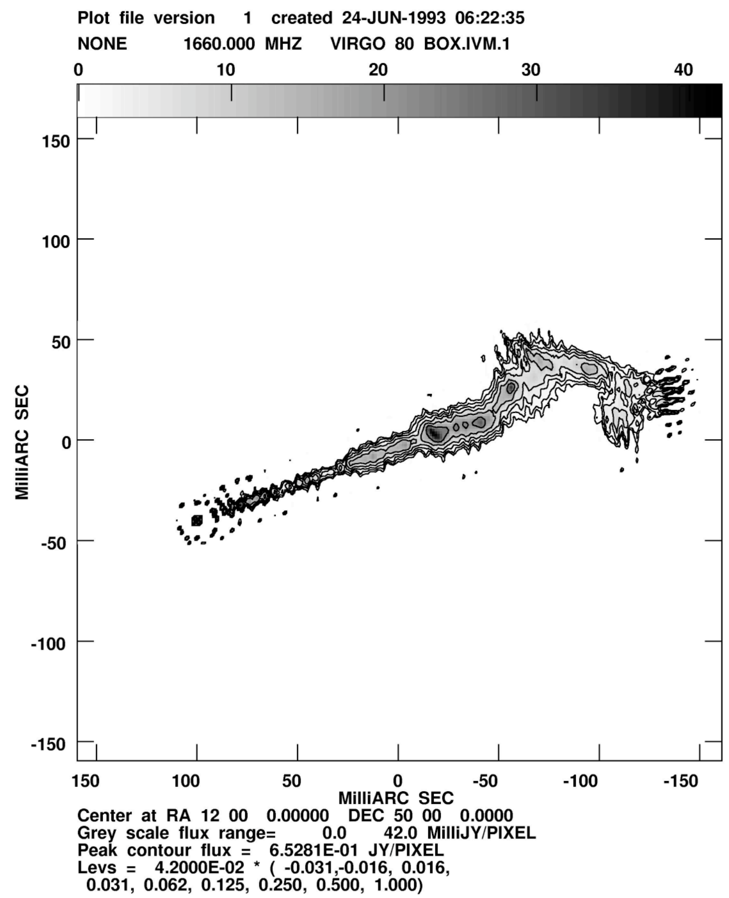
Window CLEAN: 5000 and 20000 comps



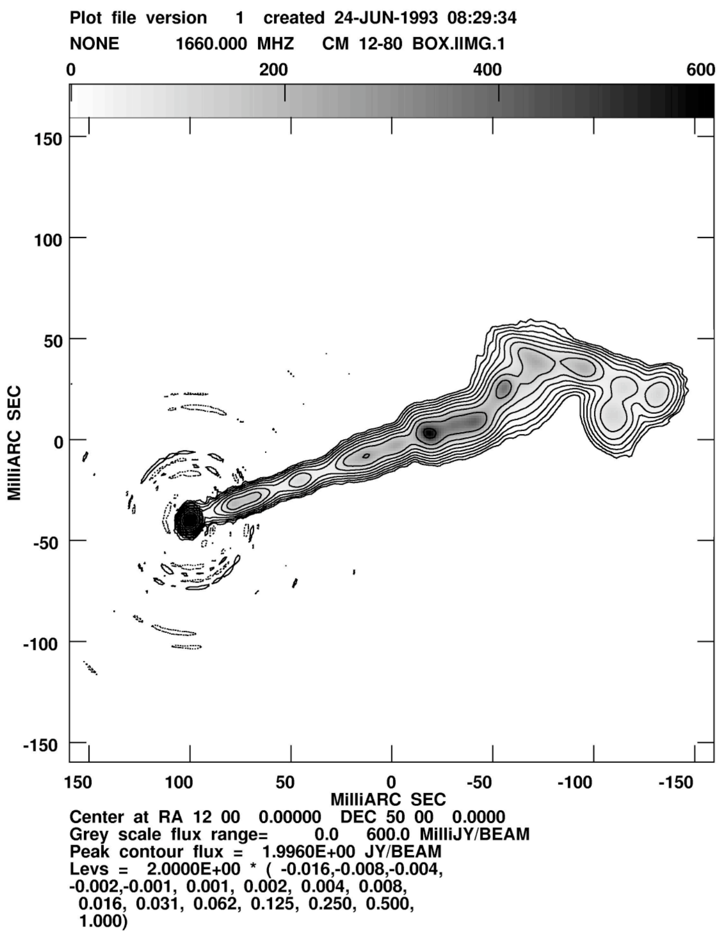


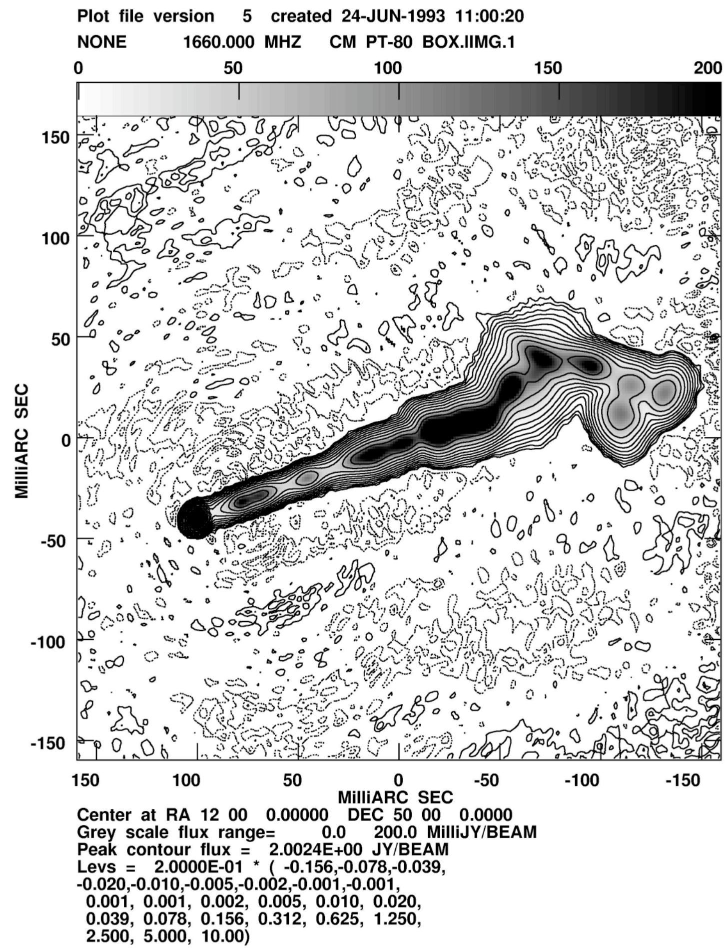
MEM: failure of super-resolution



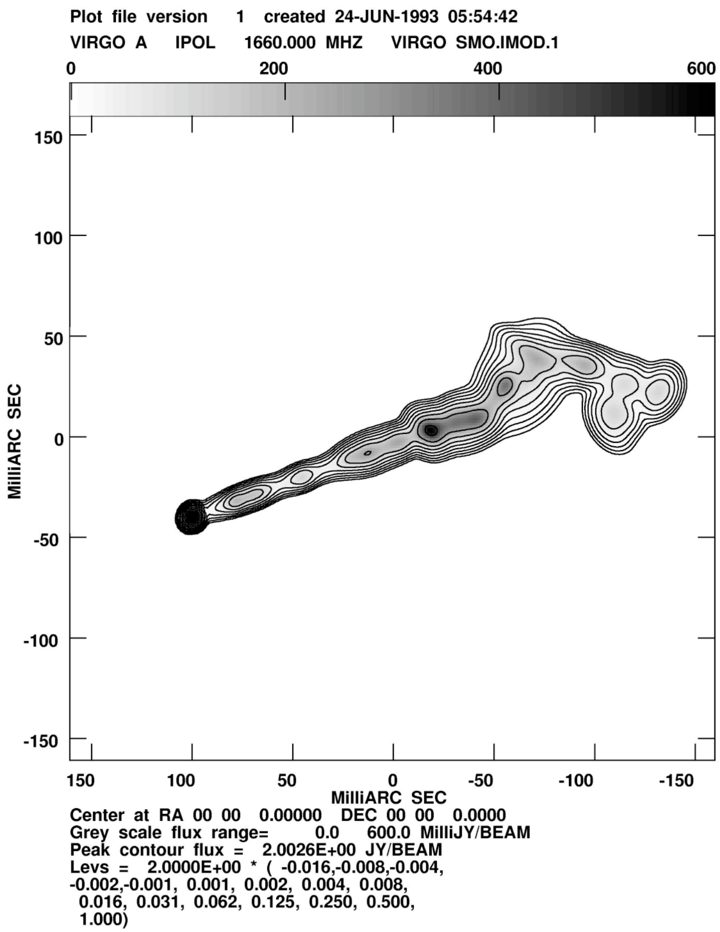


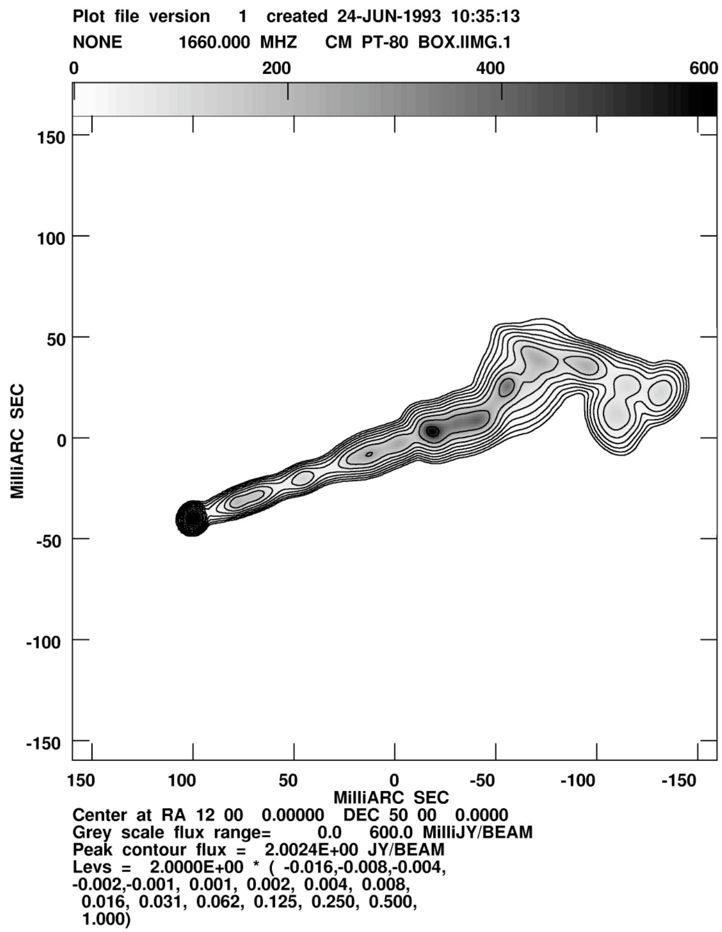
MEM: boxed, with point source removed



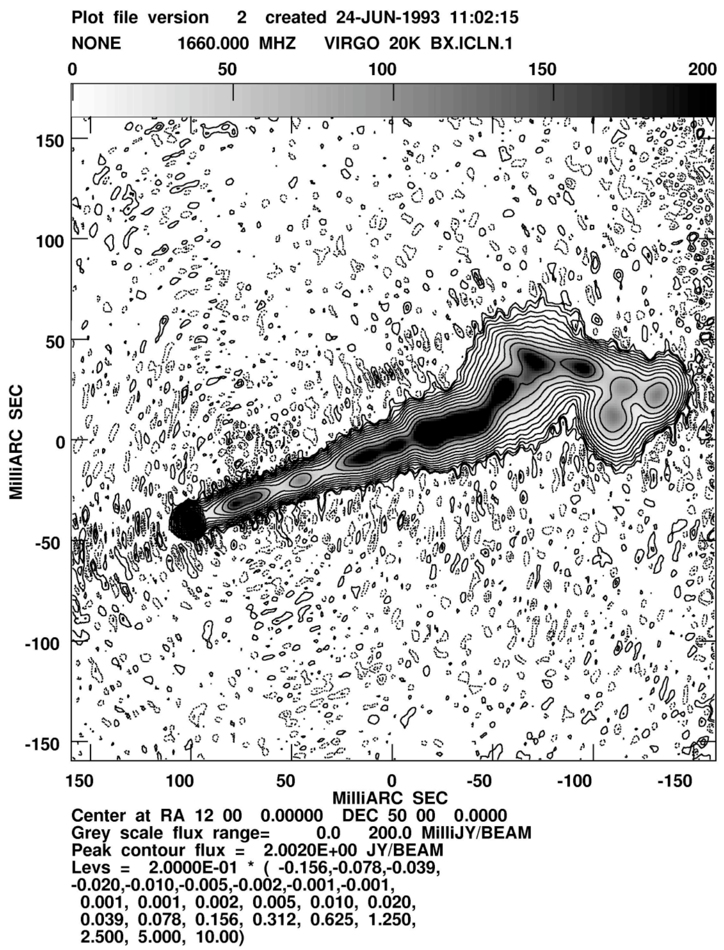


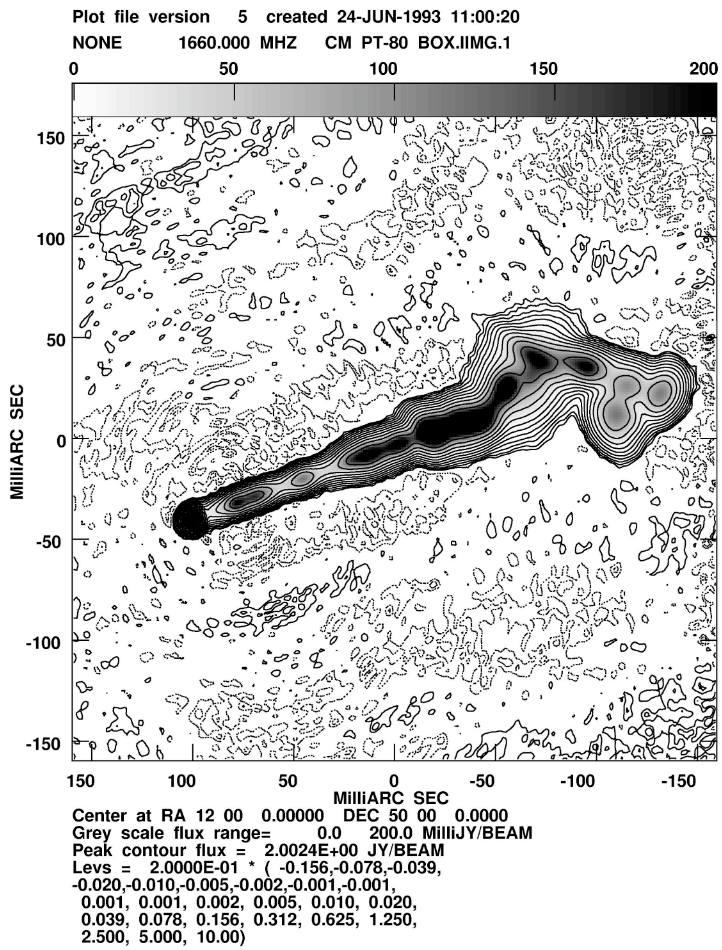
Original model and best image





Best Clean and Best MEM

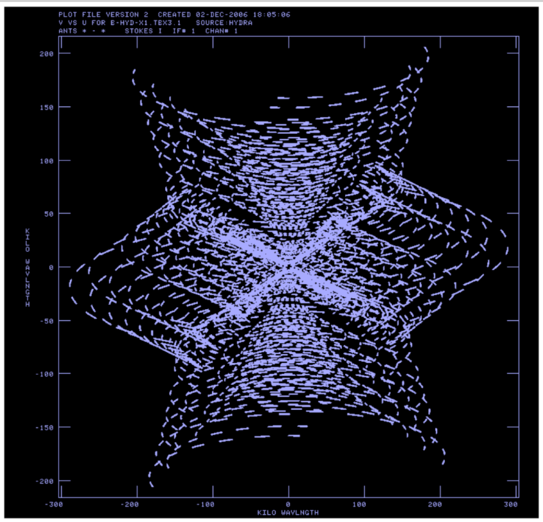


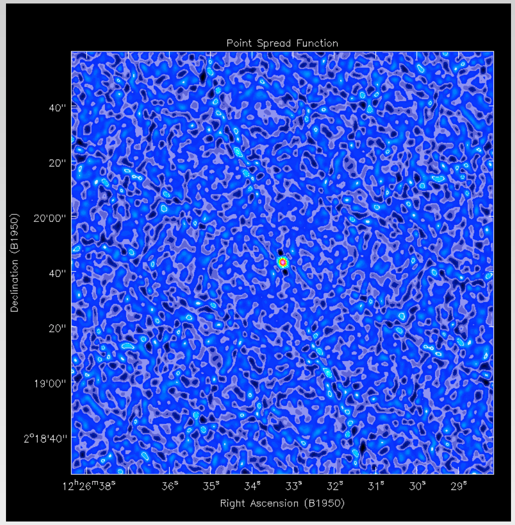


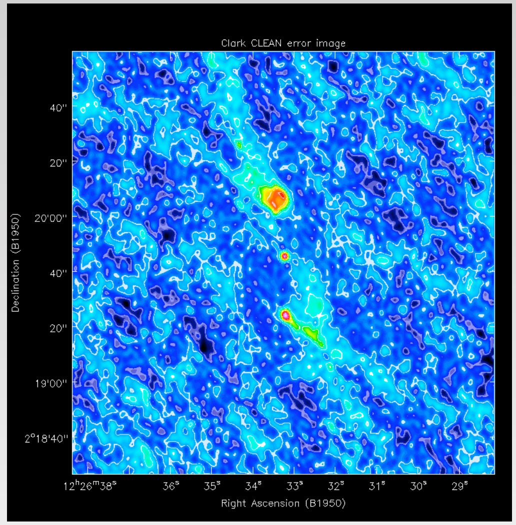
Example

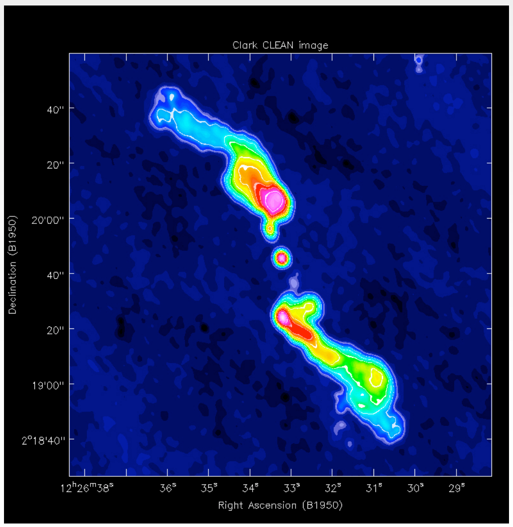
- VLA multi-snapshot observation of Hydra A-like source
- Will show
  - UV coverage
  - Visibility function
  - Point Spread Function
  - Dirty image
  - Clean images
  - Maximum Entropy images

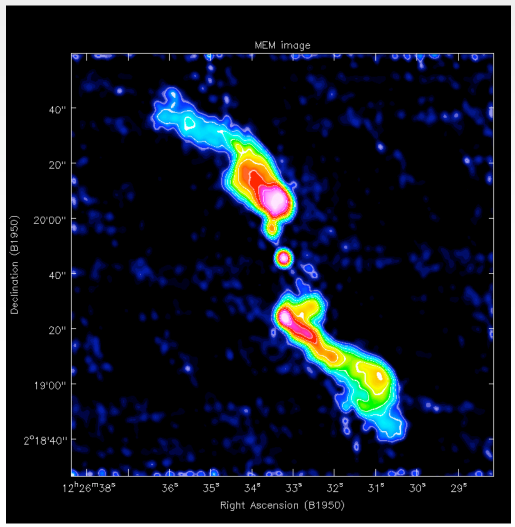
uv-coverage, PSF, Dirty image, CLEAN, MEM, Multi-scale CLEAN

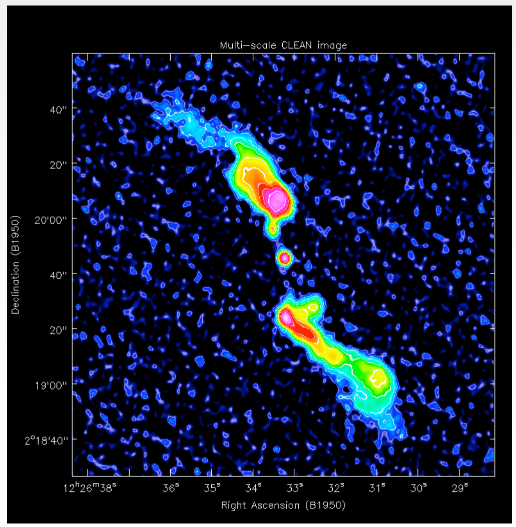












Summary

- Incomplete Fourier plane coverage leads to diffraction patterns in images
- Deconvolution algorithms can correct for these patterns
- A number of complementary algorithms exist for image deconvolution

Acknowledgements and references

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- ftp://ftp.aoc.nrao.edu/pub/dissertations/dbriggs/diss.html

Poema XIV.

Juegas todos los días con la luz del universo.
Sutil visitadora, llegas en la flor y en el agua.
Eres más que esta blanca cabecita que aprieto
como un racimo entre mis manos cada día.

(de “20 poemas de amor y una cancion desesperada” de Pablo Neruda)

Poem XIV.

Every day you play with the light of the universe.
Subtle visitor, you arrive in the flower and the water.
You are more that this white head that I hold tightly
as a cluster of fruit, every day, between my hands.

(from “20 love poems and a desperate song” by Pablo Neruda)